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Transient growth of Görtler vortices in two-dimensional compressible boundary layers. Application to surface waviness

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ABSTRACT

This work is related to the numerical calculation of the maximum spatial growth of Görtler vortices using a set of linear partial differential equations derived to take curvature effects into account. The method is based on the calculation of the most amplified perturbation using a direct/adjoint iterative procedure. A new method for computing the optimal perturbation which attain largest growth at the shortest streamwise location is presented. The resulting neutral curve is shown to restore the concept of neutral curve for the Görtler problem, defining an envelope for the unstable domain. The present paper also highlights the potential for transient growth in the development of Görtler vortices. Compressible boundary layers over curved surfaces are considered in the framework of studying the effects of surface imperfections on laminar–turbulent transition. Non-negligible values of N-factor are shown to arise over the first streamwise locations due to interactions between non-normal modes. This result emphasizes the need to take transient growth into account when predicting transition location over wavy walls.

1. Introduction

Streamwise-oriented periodic counter-rotating vortices known as Görtler vortices are present in boundary layers over concave walls. For such configurations, the flow is subject to a centrifugal instability which destabilizes the laminar boundary layer and leads to the spatially-growing, spanwise-periodic longitudinal vortices first observed by Görtler (1941). The amplitude of the vortices increases although their wavelength remains fairly constant in the spanwise direction. The inviscid mechanism, first pointed out by Rayleigh [1], arises from the disequilibrium between the centrifugal force term and the restoring normal pressure gradient and it has been proven active in both two- and three-dimensional boundary-layer flows. Practical occurrence of this phenomenon can be found in both nature and technical applications, for example over the concave part of a laminar flow wing or on the wall of a supersonic nozzle, and it is known, under some conditions, to lead the flow through transition to turbulence. Some experiments have shown that transition to turbulence in the presence of Görtler vortices can be due to a secondary instability originating from the distortion of the steady velocity profile. It has also been demonstrated that the spanwise modulations of the base flow caused by Görtler vortices can destabilize Tollmien–Schlichting waves.

A review on the topic of Görtler vortices is provided e.g. by Saric [2]: studies of boundary layers over curved walls first began with the work of Görtler [3] (1941) building upon Taylor's linear stability analysis for centrifugal instabilities in Couette flow [4]. He considered the stability of a parallel Blasius boundary layer developing over a curved wall of asymptotically large radius of curvature. Solutions of this problem were provided by Görtler and later by Hämmerlin [5] (1955) and Smith [6] (1955), but these authors noted differences in the computed neutral curves at this early stage. The solutions of the disturbance equations were found in the form of counter-rotating, streamwise-oriented vortices. Herbert [7] (1976) extended Görtler's formulation to consider finite variable radius of curvature and pointed out the dependence of the neutral curve to that curvature at low spanwise wavenumbers.

The vast majority of these early studies addressed the problem of determining the linear stability of two-dimensional boundary layers over curved surfaces under the parallel-flow approximation, i.e. assuming a base flow independent of the streamwise coordinate and thus neglecting the boundary-layer growth. Under this approximation, the partial differential linear stability equations are reduced to an ordinary differential system using a separation-of-variables method (normal mode solutions). However, for the Görtler problem, the parallel-flow approximation is no longer valid

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as opposed to the study of the Taylor instability. This has been consistently argued by Hall [8] (1982) who proved that the parallel-flow approximation is only justifiable in the limit of large spanwise wavenumbers $\beta \gg 1$ and large Görtler numbers. In this limit, he developed an asymptotic solution of the partial differential equations and obtained an expression for the upper branch of the neutral curve which was in agreement with previous numerical solutions determined for large spanwise wavenumbers. Experiments have shown that, once established, Görtler vortices remains of constant wavelength. With the development and growth of the boundary layer, the non-dimensional wavenumber scaled with the boundary layer thickness increases. The large wavenumber limit is thus ultimately appropriate far enough downstream.

A set of non-separable partial differential equations for the linear non-parallel stability problem of the Görtler vortices was first derived by Floryan and Saric [9] (1982) who performed a local stability analysis, implicitly ignoring the streamwise growth of the boundary layer. An incompressible flow with weak non-parallel effect is considered and the stability equations are classically obtained by superposing small, steady, spanwise periodic disturbances onto the base flow. An important feature of steady streamwise vortices within a shear layer is that the streamwise disturbance velocity is considered to be larger than the normal and spanwise disturbance velocities by an order of the square root of the Reynolds number. The equations are written in curvilinear coordinates, and the metric coefficients are expanded in terms of the inverse of the Reynolds number and small curvature scale. Terms of higher order than the first one with respect to curvature and Reynolds number are disregarded to obtain the parabolic system of the disturbance equation. From there, Floryan and Saric replaced partial derivatives of the vortices with respect to the streamwise coordinate by local spatial growth rates in order to reduce the system to a set of ordinary differential equations. When the growth rate vanishes, the solution of this so-called locally non-parallel stability method can be seen as a local Taylor-series solution of the full partial differential equation.

Hall [8,10] showed that the parabolic equations had to be integrated by using a downstream marching technique, as the equations for the perturbations are non-separable. An initial disturbance is imposed at a given location and its development followed as the equations are propagated downstream. Since there is no streamwise derivative of the pressure, the disturbance pressure and spanwise velocities are eliminated by cross-differentiation, leading to coupled second- and fourth-order equations in the streamwise and normal disturbance velocities respectively. He also pointed out that the existence of a neutral point strongly depends on the location and shape of the initial conditions. Hence, the concept of a unique neutral curve is not readily accessible for the Görtler problem as opposed to usual parallel flow stability problems because of the dependence to initial conditions and their location. For large spanwise wavenumbers however, the different neutral curves merge into the asymptotic and parallel-flow neutral curve. Additionally, the downstream marching technique enables the study of the transition from a concave to a convex or flat surface, which is impossible using a local analysis.

Day et al. [11] (1990) and later Goulpié et al. [12] (1996) have made detailed comparisons between the normal-mode approach, employed in particular by Floryan and Saric, and the approach consisting of marching the equations downstream developed by Hall. The results obtained with the locally non-parallel method can be interpreted as the limit the marching solutions asymptotically tend to some distance away from the leading-edge. Differences are found in the determination of the first neutral point, before the agreement between marching and normal-mode solutions is obtained at some distance from the leading-edge. If the initial conditions are taken as solutions of the local problem, a good agreement is obtained between both methods when comparing neutral curves. However, the use of particular initial conditions such as the initial disturbances employed by Hall can lead to wide discrepancies in the determination of the neutral stability, a result also emphasized by Kalburgi et al. [13] (1998). Bottaro and Luchini (1999) [14] recently came back to the local stability theory for the Görtler problem by comparing local solutions at various order in terms of the inverse of the Görtler number (including, at leading order, previous local theories, as e.g. the one of Görtler) with solutions obtained with the marching technique. They found good agreement between both methods far enough from the leading-edge when the initial conditions are prescribed.

Luchini and Bottaro [15] (1998) later studied the receptivity of Görtler vortices to both free stream perturbations and wall disturbances. They computed the Green's functions that lead to the most amplified Görtler vortices when multiplying external disturbances. The Green's functions are obtained from a backward-in-time integration of the adjoint parabolic system. Cossu et al. [16] (2000) first employed an optimal approach to compute the maximum spatial growth rate of Görtler vortices over a concave wall. Their method is based on the computation of a discrete approximation of the spatial propagator which relates the downstream response to the perturbation at the inlet. In particular, the similarities of the inlet optimal perturbations and their resulting responses between the Görtler problem and the case of a flat plate (corresponding to a Görtler number set to zero) studied by Andersson et al. [17] and Luchini [18] are emphasized. Recent computations by means of direct numerical simulation (DNS) by Schrader et al. [19] (2011) compared the generation of Görtler vortices by localized roughness and vortical modes. The latter are more efficient as the boundary layer is found to be most receptive to zero- and low-frequency free-stream vortices which excite steady or slowly travelling Görtler vortices. The connection between transient growth and exponential amplification is also pointed out, with rapid non-modal growth taking place near the leading-edge and the evolution of boundary layer streaks into Görtler vortices.

The present work aims at investigating the spatial growth of Görtler vortices using an optimal perturbation approach. Accurate determination of perturbation amplification, especially close to the leading-edge, is tackled by taking transient growth into account. The boundary layer receptivity is also investigated. The method is then applied to the problem of transition prediction over wavy surfaces. In Section 2, the appropriate linear partial differential equations for a boundary-layer flow over a curved wall are derived. The optimization procedure related to the computation of the optimal perturbation is detailed. The optimization problem is solved using iterations of the direct/adjoint system. The optimal perturbation which attains largest growth at the shortest chordwise location is then determined using the first neutral position as the location where the gain is maximized. Section 3.1 presents a receptivity analysis for an incompressible boundary layers over a concave wall. The most dangerous inlet location is also determined. The optimal perturbation method proves adequate to compute the neutral curve for the Görtler problem, delimiting an envelope for the unstable domain. Section 3.2 highlights the potential for transient growth in the early development of Görtler vortices. Such transient growth provides non-negligible initial N factor values which have to be taken into account when predicting laminar/turbulent transition. In relation to the study of transition in the presence of surface imperfections, the practical case of wavy surfaces is considered in Section 4. N factor curves for flows over such configurations are computed, taking the initial phase of transient growth into account. As real situations imply compressible flows, the last section also presents N factor computations at various Mach numbers for compressible boundary layers over wavy walls.