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Experimental and numerical parametric analysis of a reoriented duct flow

Ozge Baskan[a,*], Michel F.M. Speetjens[b], Guy Metcalfe[c,d], Herman J.H. Clercx[a]

[a] *Fluid Dynamics Laboratory, Department of Applied Physics, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands*

[b] *Energy Technology Laboratory, Department of Mechanical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands*

[c] *Commonwealth Scientific and Industrial Research Organisation (CSIRO), Clayton VIC 3169, Australia*

[d] *Swinburne University of Technology, Hawthorn VIC 3122, Australia*

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This study concerns a coupled experimental–numerical analysis of scalar transport in reoriented duct flows found in industrial mixing processes. To this end the study adopts the Rotated Arc Mixer (RAM) as the representative configuration. The focus is on the effects of geometrical (i.e. reorientation angle Θ) and temporal (i.e. reorientation frequency τ) parameters of generic inline mixing devices on the Lagrangian particle dynamics and scalar field evolution. Lagrangian dynamics are investigated by constructing Poincaré sections from analytic flow solutions and stroboscopic measurements of particle positions in 2D RAM laboratory setup. In order to obtain the optimal mixing and homogenization of scalar fields, dye visualizations are performed for an extensive set of parameters. The mixing quality in parameter space is quantitatively evaluated by means of the intensity of segregation. These results are used to determine the optimum forcing protocol. The outcome of this study validates the qualitative agreement in mixing characteristics of 2D time-periodic and 3D spatially-periodic flows and confirms the good mixing performance found before for certain RAM configurations. Moreover, we demonstrate that even more efficient protocols can be devised by suitably tuning the sequence of the reorientation angle. This knowledge might eventually lead to optimized 3D reoriented duct flow mixers.

1. Introduction

Scalar advection in reoriented duct flows is an active area of research due to their particular relevance to continuous industrial mixers such as the Kenics mixer [1–3], SMX mixer [1,3] and the Rotated Arc Mixer [4], to microfluidic lab-on-a-chip diagnostic devices, and to the bulk manufacturing of fine chemicals. The flow in these configurations can be characterized by three common features: (i) a continuous throughflow; (ii) a transverse flow generated by internal elements or moving boundaries; (iii) systematic reorientation of the flow along the duct axis [5]. The interplay between the axial and transverse flow, which is controlled by a set of parameters, determines the state of the mixing. In this respect, it is of importance to deepen insight into the effects of these control parameters on the mixing efficiency.

This study adopts a 2D simplified Rotated Arc Mixer (RAM) as a representative configuration. The simplification relies on that the downstream progression of reoriented duct flows in axial direction is dynamically similar to evolution in time, implying 3D spatially-periodic flows can be well represented by 2D time-periodic flows. This approach is common in the analyses of reoriented duct flows [6,5,7] and leans on the fact that Lagrangian transport in a 3D reoriented duct flow is governed by Hamiltonian mechanisms of a 2D time-periodic flow [8]. Likewise, the 3D steady scalar transport in spatially-periodic systems can be transformed into 2D time-dependent systems assuming that the axial diffusion is negligible [9]. In a practical context, the simplified cases retain the fundamental kinematic properties and deviate from the real 3D cases quantitatively, which validates the suitability of the simplified approaches for qualitative mixing analyses [8] (including parameter studies).

In this context, this study aims to experimentally investigate and validate the generic Hamiltonian dynamics governing the transport in reoriented duct flows. For this reason, Poincaré sectioning (which is a frequently-used dynamical system tool to

* Corresponding author.
E-mail address: o.baskan@tue.nl (O. Baskan).

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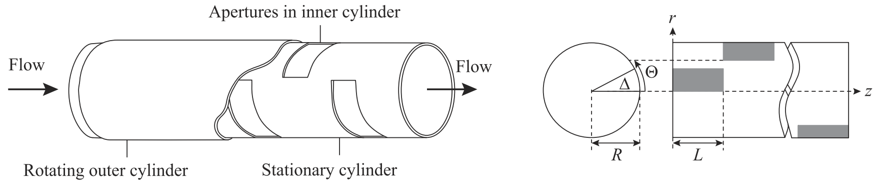

Fig. 1. A schematic of the Rotated Arc Mixer (RAM). The left panel shows the geometry of the RAM with the rotating outer cylinder, stationary inner cylinder, and the axial throughflow. The right panel shows the definition of the cylindrical coordinate system, the cylinder radius R, offset angle Θ, aperture arc angle Δ, and the aperture length L. [10].

visualize the qualitative properties of a time-dependent Hamiltonian system) is used to analyze the Lagrangian flow topology. The topology of the flow is dependent on two parameters controlling the flow: reorientation angle and reorientation frequency. Hence, the focus of this study is on the effect of these parameters on the transport properties and the approach is a coupled experimental and numerical method, which is composed of three steps: (i) the investigation of the Lagrangian flow topology by means of the analytical Stokes flow solution of the 2D RAM flow; (ii) the experimental analysis of the Lagrangian flow topology via direct measurements of Poincaré sections; (iii) the experimental investigation of the scalar transport via dye visualizations. The analytical solution is used to assess the characteristics of the flow topology in a general manner in a wide parameter space and thus enables construction of a regime diagram. Two complementary methods, Poincaré sections and dye visualizations, are employed experimentally to deepen insight into the Lagrangian transport. The advantage of the experiments over the numerical simulations is that long-term evolution of the scalar fields can be investigated; numerically this is very challenging, if not impossible, due to the complex structure of the scalar field patterns. The numerical simulations of such complex patterns necessitate the use of sufficiently fine mesh elements to keep numerical diffusion at a level such that (any part of) the scalar pattern is not diffused artificially. The use of a sufficiently fine mesh may not always be possible due to the limitations in computational resources. Especially, in the limit of zero-diffusivity, which is the case in the present study, the numerical simulations become unfeasible.

The advantage of simplification from 3D steady flow to 2D time-periodic flow, from the experimental point of view, is the total number of periods that can be achieved during an experiment and the direct visualization of experimental Poincaré sections. Reliable Poincaré sections can be constructed with the data of at least hundreds of periods. For 2D flow case, this is achieved by running the system for the desired number of periods without any constraint in space and Poincaré sections can be directly measured without any additional effort, e.g. the integration of the flow field. Yet, if one wants to acquire the same information for the 3D case, this necessitates the construction of a setup with hundreds of periodic segments of the real 3D mixing device, which is not very practical. As Lagrangian dynamics of 3D reoriented duct flows are analogous to Hamiltonian dynamics of a 2D time-periodic flow, the simplification from 3D steady flow to 2D time-periodic flow retains the topological characteristics which are generic to all 3D reoriented duct flows [8]. Hence, the simplified analysis presented in this study provides valuable qualitative information that can be used to enhance the mixing quality in realistic mixing systems.

2. Problem definition

The advective–diffusive scalar transport in an incompressible flow is governed by

$$\frac{dC}{dt} = \frac{\partial C}{\partial t} + \boldsymbol{u} \cdot \nabla C = \frac{1}{Pe}\nabla^2 C, \tag{1}$$

with appropriate initial and boundary conditions, where all variables are dimensionless and C is the scalar field, $\boldsymbol{u}$ the flow field, t the time and $Pe = UR/D$ the Peclét number with U the characteristic velocity, R the characteristic length and D the material diffusivity. When Pe tends to infinity, i.e. in the limit of zero-diffusivity, the material derivative dC/dt tends to zero meaning that the tracers are advected passively.

Fig. 1 shows the 3D RAM geometry. It consists of two concentric cylinders; a stationary inner duct with consecutive windows that are offset in angular direction and an outer rotating cylinder that induces a transverse flow via viscous drag at the windows as the flow progresses axially with a constant flow rate. A duct segment bearing a window is called a cell and each cell has an identical flow called base flow. Each successive cell has an identical axial length L and is rotated about the duct axis by an angle Θ, which represents the geometrical control parameter. In the simplest case Θ is fixed. The temporal control parameter of the 3D RAM is the ratio of axial to transversal time scales $\beta = \Omega L/U_{\text{mean}}$, where Ω is the angular velocity of the rotating outer cylinder and U_{mean} the mean axial velocity. In the 2D case, β simplifies to $\tau = \Omega T = T/T_f$, where T is the dimensionful time of one aperture activation and $T_f = R/U$ the typical flow time scale with U the velocity of arcs. In the current study, the experiments are performed for a total of eight Θ values and five τ values for each Θ case. 2D time-periodic flow is accomplished by successive activation of p arcs, resulting in $\mathcal{T} = p\tau$ as total period time, where offset Θ is chosen such that $p\Theta = m2\pi$ with m an integer. The full parameter set consist of parameters $\Theta, \tau, Pe = \Omega R^2/D$ and $Re = \Omega R^2/\nu$, where Re is the Reynolds number and ν the kinematic viscosity. However, investigation of the effects of Re and Pe is beyond the scope of this study which is limited to the flow regime at the Stokes limit ($Re = 0$) and the scalar transport in the advective limit ($Pe^{-1} \approx 0$). Theoretical studies of non-zero Re and finite Pe on the RAM flow are given in the works by Speetjens et al. [8] and Lester et al. [11], respectively.

The tracer dynamics in 2D time-periodic flows, described by their current position $\boldsymbol{x}(t)$ and released at initial position $\boldsymbol{x}_0$, is governed by the Hamiltonian equations of motion

$$\frac{dx}{dt} = \frac{\partial H}{\partial y}, \qquad \frac{dy}{dt} = -\frac{\partial H}{\partial x}, \tag{2}$$

with $H(x, y, t) = H(x, y, t + \mathcal{T})$ the Hamiltonian [12], where H is analogous to the stream function $\psi(x, y)$ of the flow. The Hamiltonian of the 2D RAM flow, the stream function of which is given in Hwu et al. [13], can be arc-wise constructed (in polar coordinates) via $H(r, \theta)|_{\text{arc n}} = \psi(r, \theta - (n-1)\Theta)$ for $1 \le n \le p$ and can be visualized by Poincaré section. Poincaré sectioning is a frequently-used dynamical system tool to analyze the mixing quality of a flow by determining its chaotic and non-chaotic regions. It is the set of the tracer positions acquired at subsequent time levels, i.e. the set of tracer positions $\boldsymbol{X}(\boldsymbol{x}_0) = \{\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots\}$, with $\boldsymbol{x}_k = \boldsymbol{x}(k\mathcal{T})$ the position after k periods.

When $\mathcal{T} \to 0$, the Hamiltonian is the average of the arc-wise stream functions, i.e. $H(r, \theta) = \frac{1}{p}\sum_{n=1}^{p} \psi(r, \theta - (n-1)\Theta)$. This corresponds with the steady flow created by the simultaneous