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Sound propagation through a binary mixture of rarefied gases at arbitrary sound frequency

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HIGHLIGHTS

- Numerical solution of the McCormack model to the linearized Boltzmann equation.
- Influence of the sound frequency on the macrocharacteristics of binary mixtures.
- Influence of the molecular mass ratio of species on the sound propagation.
- Deviation of the macrocharacteristics of the mixture from those for a single gas.

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ABSTRACT

The sound propagation through a binary mixture of rarefied gases in a semi-infinite space is investigated on the basis of the McCormack model to the linearized Boltzmann equation. The source of sound waves is an infinite and flat plate oscillating in the direction normal to its own plane. It is considered a fully established oscillation so that the solution of the kinetic equation depends on time harmonically. The diffuse scattering gas–surface interaction law is assumed as the boundary condition on the oscillatory plate. A realistic intermolecular potential based on experimental data for the transport coefficients of the mixture is employed in the calculations. Two mixtures of noble gases, namely, Helium–Argon and Helium–Xenon, are considered in order to investigate the influence of the molecular mass ratio of species on the problem. A wide range of the oscillation speed parameter, defined as the ratio of intermolecular collision frequency to sound frequency, is considered. The amplitudes and phases of the macrocharacteristics of the gas flow are calculated and the results are compared to those obtained for a single gas.

1. Introduction

The classical theory of sound propagation through a binary mixture of gases is based on the continuum mechanics equations, namely, Navier–Stokes, Fourier and Fick equations. However, as already pointed out in our previous works [1,2], these equations are valid under two conditions: (i) the characteristic length of the gas flow domain must be significantly larger than the molecular mean free path and (ii) the characteristic time of the gas flow must be significantly larger than the mean free time. Moreover, the classical theory predicts that a sound wave is characterized by a phase speed and attenuation coefficient constant over the space. However, this assumption is valid only far from the source of sound waves, i.e. outside of the Knudsen layer which has the order of a mean free path. The results presented in Refs. [1,2] show that the wave characteristics near the source are significantly different from those far from the surface even at low frequencies, i.e. the solution is not harmonic in space as predicted by the classical theory.

To characterize the speed of oscillation the so-called oscillation speed parameter, defined as the ratio of the intermolecular collision frequency to the sound frequency, is introduced. According to this parameter, three oscillation regimes can be established for the gaseous mixture flow induced by the sound propagation through it. First, the low oscillation speed regime, in which there are many intermolecular collisions occurring during one cycle of the oscillation. In this regime the classical theory is valid to describe the sound characteristics far from the source. Second, the high frequency oscillation regime in which there are few intermolecular collisions during one cycle of the oscillation. In this regime the

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intermolecular collisions can be neglected and the problem can be solved analytically on the level of the distribution function for each species of the mixture. Third, the transitional regime in which both collision and oscillation frequencies have the same order. In such a regime the problem must be solved on the microscopic level with basis of the distribution function of molecular velocities. Then, an approach based on the solution of a system of coupled non-stationary Boltzmann equations for each species of the mixture can be used in the transitional regime.

The correct description of sound propagation through a rarefied gaseous mixture for arbitrary sound frequencies is very important in many fields. For instance, in the development and performance improvement of MEMS (Micro-Electro-Mechanical Systems) with some oscillating pieces surrounded by a gaseous mixture such as air, acoustic mixture separators, vacuum industry, etc. Moreover, since a sound wave can also be caused by a temperature oscillation, the understanding of the transfer processes in the propagation of sound through a gaseous mixture is very important to the development of thermoacoustic devices [3] such as small engines designed to convert heat to sound and then to electricity. Depending on the gas mixture under consideration and on the concentration of species, the behavior of the gaseous mixture flow caused by the sound propagation through it can be very different from that corresponding to a single gas.

In spite of the great importance for the development of new technologies and the understanding of some phenomena such as acoustic gas separation, there are few papers in the literature concerning the subject under investigation in the present work. Theoretically, the majority of papers available in the literature for gaseous mixtures use the assumption of plane harmonic waves in space, see e.g. Refs. [4,5], and such an assumption is not always valid. In Ref. [4], the coupled linearized Boltzmann equations for a binary mixture of monatomic gases were solved as a perturbed eigenvalue problem. The absorption and attenuation coefficients for a Helium–Argon mixture were calculated for the special case of Maxwell molecules. In Ref. [5], a kinetic model to the Boltzmann equation corresponding to a two-fluid hydrodynamic theory was employed. The absorption and attenuation coefficients were obtained for the mixtures Helium–Argon and Helium–Xenon assuming that the gaseous particles interact according to the Lennard-Jones potential. A damping force exerted by a binary gaseous mixture on a vibrating plate was calculated in Ref. [6] with basis on a kinetic model for the linearized Boltzmann equation. Experimentally, there is lack of data to compare to theoretical results and few experimental results reported in some papers are restricted to low sound frequencies, see e.g. [7].

In the present work the sound propagation through a binary mixture of monatomic gases is studied on the basis of the kinetic model to the linearized Boltzmann equation. We are going to calculate the amplitudes and phases of the macrocharacteristics of the gaseous mixture such as bulk velocity, density and temperature deviations from equilibrium state, as function of both oscillation speed parameter and molar fraction of the mixture. A realistic intermolecular interaction law [8] based on experimental data for the transport coefficients of the mixture is employed, as well as the diffuse scattering gas–surface interaction law. Moreover, in order to analyze the influence of the molecular mass ratio of species on the solution of the problem, two mixtures of noble gases are considered, namely, Helium–Argon (He–Ar) and Helium–Xenon (He–Xe). The results are compared to those corresponding to a single gas.

2. Statement of the problem

Consider a binary mixture of monatomic gases in a semi-infinite space $x' > 0$. An infinite flat plate located at $x' = 0$, and parallel to the plane yz, oscillates harmonically in the x' direction with a frequency ω so that its velocity depends on the time t as

$$U_w(t) = \Re[U_m \exp(-i\omega t)], \tag{1}$$

where $\Re$ denotes the real part of the complex quantity and i is the imaginary unit. U_m is the velocity amplitude of the plate, which is assumed as to be very small when compared with the characteristic molecular speed v_m of the gaseous mixture, i.e.

$$U_m \ll v_m, \quad v_m = \sqrt{\frac{2k_B T_0}{m}}, \tag{2}$$

where k_B is the Boltzmann constant and T_0 is the temperature of the mixture in equilibrium state. The mean molecular mass m of the mixture is defined as

$$m = C_0 m_1 + (1 - C_0) m_2, \tag{3}$$

where

$$C_0 = \frac{n_{01}}{n_{01} + n_{02}}, \tag{4}$$

is the molar fraction of the mixture in equilibrium, $n_{0\alpha}$ and m_α ($\alpha = 1, 2$) denote the equilibrium number densities and molecular masses of species α.

The sound waves generated by the oscillating plate propagate through the gaseous mixture in the x'-direction and disturb the equilibrium properties of the mixture such as number density n_0 and temperature T_0. Therefore, the disturbed gaseous mixture is characterized by a density $n(t, x')$ and temperature $T(t, x')$. Moreover, the bulk velocity $U(t, x')$ and heat flux $Q(t, x')$ in the x'-direction appear. In practice, the only quantity measured is the deviation of pressure in the direction of sound propagation, viz. the difference $P_{xx} - p_0$, where $p_0 = n_0 k_B T_0$ denotes the pressure of the mixture in the equilibrium state. The calculation of the diagonal terms of a pressure tensor allows us to determine the pressure difference whose amplitude and phase can be measured. Note that the pressure of the mixture is given as

$$p = \frac{1}{3}(P_{xx} + P_{yy} + P_{zz}). \tag{5}$$

Since the condition of isotropy in the plane yz is considered, then

$$P_{yy} = P_{zz}. \tag{6}$$

For further derivations, it is more convenient to introduce the dimensionless x-coordinate as

$$x = \frac{\omega}{v_m} x', \tag{7}$$

where v_m/ω corresponds to the average distance traveled by gaseous particles during one cycle of the oscillation.

The gas flow induced by the sound propagation through the gaseous mixture is considered as fully established. Consequently, all the quantities which describe the behavior of the mixture, i.e. density, temperature, pressure, bulk velocity and heat flux, depend on time harmonically. In order to calculate these macroscopic quantities, dimensionless complex functions are introduced as

$$\Re[\varrho(x)e^{-i\omega t}] = \frac{n(t, x) - n_0}{n_0}\frac{v_m}{U_m}, \tag{8}$$

$$\Re[\tau(x)e^{-i\omega t}] = \frac{T(t, x) - T_0}{T_0}\frac{v_m}{U_m}, \tag{9}$$

$$\Re[\Pi(x)e^{-i\omega t}] = \frac{P_{xx}(t, x) - p_0}{2p_0}\frac{v_m}{U_m}, \tag{10}$$

$$\Re[u(x)e^{-i\omega t}] = \frac{U(t, x)}{U_m}, \tag{11}$$

$$\Re[q(x)e^{-i\omega t}] = \frac{Q(t, x)}{p_0 U_m}. \tag{12}$$