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Development and validation of a non-linear spectral model for water waves over variable depth

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ABSTRACT

In the present paper two numerical schemes for propagating waves over a variable bathymetry in an existing High-Order Spectral (HOS) model are introduced. The first scheme was first developed by Liu and Yue (1998), and the second one is an improved scheme which considers two independent orders of non-linearity: one for the bottom and one for the free-surface elevation. We investigate the numerical properties (accuracy, convergence rate, efficiency) of both schemes with respect to the numerical parameters on a simple configuration. To validate the proposed schemes, we first consider Bragg reflection from a sinusoidal bottom patch — as an example of a small bottom variation around a mean water depth. The second validation case focuses on a larger bottom variation with the study of the shoaling of linear waves. Finally, an application is performed to demonstrate the applicability of the proposed model to non-linear cases where the bottom variation is important. In this concern, the very well documented test case of a 2D underwater bar is studied in detail. Comparisons are provided with both experimental and numerical results.

1. Introduction

The accurate modelling of surface gravity waves over non-negligible bottom topography is of major interest in ocean engineering, especially in the field of marine renewable systems. These marine structures are intended to be deployed in limited water depth, where the effect of variable bathymetry on local wave conditions is very significant. Indeed, when entering shallow water zones, waves are strongly affected by the bottom through shoaling, refraction, diffraction, reflection and the resulting variations in local wave speed. Thus, the accurate description of the wave field over variable depth is a prerequisite for the accurate prediction of wave loads acting on structures in coastal zones.

For this purpose, a wide variety of non-linear flow models have been developed during the last decades. Some of them are based on the solution of the Reynolds Averaged Navier–Stokes equations, such as the CFD models presented in [1], but the computational effort with these models remains prohibitive. Thus, most of the non-linear flow models for wave propagation were developed in the framework of the potential flow theory, considering that the propagation in the ocean is mostly irrotational and inviscid (neglecting wave breaking at the sea surface and dissipation due to bottom friction).

The Boundary Element Model (BEM) (see for instance [2]) is one of the methods used to represent wave propagation over non-uniform depth in wide domains. The problem is solved on the boundaries, allowing the reduction of the problem size. Moreover, it is a very flexible method as it can account for a variable bottom and very complex geometries, including structures. Nevertheless, it requires the inversion of full matrices, reducing the efficiency of the method. Some recent developments using the Fast Multiple Acceleration technique (see [3,4]) intend to improve the efficiency of the BEM model. Finite difference methods (see [5]) are also flexible in terms of geometries, but as any volume-type method it requires a high number of nodes to represent the whole computational domain. However, this formulation leads to sparse matrices allowing the use of advanced numerical procedures and resulting in a good efficiency. Other methods for modelling wave propagation over variable depth can also be found in the literature such as Finite Element Methods [6,7] or a recent fully dispersive coupled-mode model described by Belibassakis et al. [8]. Boussinesq methods were initially developed for small relative water depths in the framework of weakly non-linear waves,

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but the last developments of high order versions of Boussinesq approximations developed by Bingham et al. [9] and Madsen et al. [10] allow to account for larger water depths at little extra computational cost which remains quite attractive.

Interesting properties of spectral methods in terms of convergence have led to the development of numerous models. We can first cite the Direct Method, introduced by Fenton and Rienecker [11], which solves the problem on the free-surface (free-surface method) allowing a reduction of one dimension. Nevertheless, the required inversion of a fully populated matrix makes it not very efficient. The pseudo-spectral σ-transform model introduced by Chern et al. [12] allows the modelling of more complex geometries but still with a high computational time due to the necessity to discretize the whole fluid domain. The pseudo-integral/spectral method improved by Fructus et al. [13] uses a pseudo-spectral solution added to an integral solution to cope with steeper cases and variable bathymetries. The DNO (Dirichlet to Neumann Operator) method was initiated by Craig and Sulem [14] for a flat bottom. This method was extended to a variable bottom by Guyenne and Nicholls [15] and Craig et al. [16] by introducing another operator depending only on the variation of the bottom. It was then improved numerically by Guyenne and Nicholls [17].

In the present paper, we use the High-Order Spectral (HOS) method. This non-linear potential method has been initially developed by West et al. [18] and Dommermuth and Yue [19] for a flat bottom, and extensively validated for different configurations in the LHEEA Lab., from regular waves up to irregular multidirectional wavefields (see [20,21]). This model, named HOS-ocean is available as an open-source version.[1] The HOS formalism presents expansions identical to the DNO method, as demonstrated in [22], and several advantages. First of all, the problem is formulated on the free surface, allowing a reduction of one dimension when solving the problem. Moreover, it allows the solution of the problem with the fully non-linear free surface boundary conditions, it shows excellent convergence properties and it has a low computational cost. Therefore this method is very efficient and accurate, but is initially limited to simple geometries in both horizontal and vertical directions. Non-linear potential flow models cited above consider a varying bottom but few of them were based on the HOS scheme. As demonstrated in [23], the HOS model appears more efficient than the most advanced potential flow solvers, when compared for wave propagation on uniform depth. Thus it appears interesting to extend such a model to a variable bathymetry in order to broaden its application range, while keeping the numerical efficiency. Liu and Yue [24] provided one study with the HOS method which takes into account a variable bathymetry, and presented one simulation case but limited to a small variation of the bottom. The first scheme used in the present paper is based on their work. The second scheme is an improvement of the first one by considering two different orders of non-linearity: one for the bottom and one for the free-surface. This dissociation of the orders of non-linearity was presented with the DNO method in [17], and will be adapted here to the HOS formalism.

The two HOS methods presented (called original method and improved method) are explained in detail in the present paper. Some of the work with the original method presented hereafter has already been partially introduced in [25]. For documenting the accuracy of both schemes, a test case with a constant variation of the bottom is computed to check the convergence on a simple geometry with a non-linear reference solution. It is a highly demanding test case because a constant but wrong depth is imposed in the whole domain. Then both methods are applied to 2D monochromatic cases. Bragg reflection from a sinusoidal bottom patch will first be considered, as an example of a small bottom variation around a mean water depth. To validate our models with larger bottom variation, the shoaling of linear waves is studied in the second case. Finally, two well-documented application cases (both numerically and experimentally) are provided. We validate and compare the two methods for realistic and large bottom variations. The first case considers the transformation of a non-linear, monochromatic wave as it travels up and over a submerged bar with a mild slope. This validation case has often been used as a discriminating test case for non-linear models of surface waves propagation over a variable bottom because it requires the accurate propagation of waves in both deep and shallow water. A comparison to the experimental data and to other numerical results is provided, as well as a comparison of both methods. The second application case considers the same experimental set-up but with a steeper slope, and proves the ability of our models to represent cases with strong variations of the bathymetry and large bottom gradients.

[1] https://github.com/LHEEA/HOS-ocean/wiki.

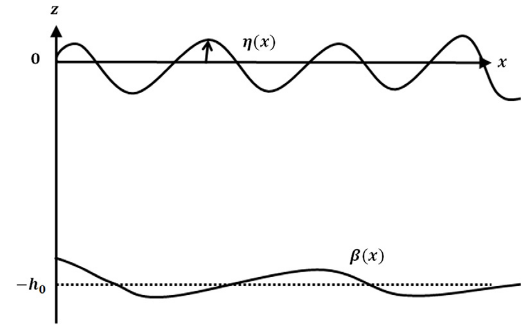

Fig. 1. Description of the fluid domain.

2. Methods and algorithms

2.1. Hypothesis and formulation of the problem

A 2D rectangular fluid domain and a Cartesian coordinate system with the origin O located at one corner of the domain are considered. The z axis is vertical and oriented upwards, with the level $z = 0$ corresponding to the mean water level. $z = \eta(x, t)$ represents the free surface elevation, h the total water depth, h_0 the mean depth and $\beta(x)$ the bottom variation, such as $-h(x) = -h_0 + \beta(x)$. Thus, the domain considered is: $-h_0 + \beta(x) \le z \le \eta(x)$ (see Fig. 1).

An infinite domain assumption is adopted with the assumption of periodic boundary conditions in the horizontal plane.

A potential flow formalism is used (incompressible and inviscid fluid, irrotational flow). Given these assumptions, the velocity $\vec{V}$ derives from a potential ϕ: $\vec{V}(x, z, t) = \vec{\nabla}\phi$ and the continuity equation becomes the Laplace equation in the fluid domain D:

$$\Delta\phi = 0. \tag{1}$$

Following Zakharov [26], both kinematic and dynamic non-linear free-surface boundary conditions (FSBC) are written in terms of surface quantities η and $\tilde{\phi}$ ($\tilde{\phi}(x, t) = \phi(x, z = \eta, t)$), and expressed at the exact free-surface position $z = \eta$:

$$\frac{\partial\eta}{\partial t} = \left(1 + \left|\frac{\partial\eta}{\partial x}\right|^2\right)\frac{\partial\phi}{\partial z} - \frac{\partial\tilde{\phi}}{\partial x}.\frac{\partial\eta}{\partial x} \tag{2}$$

$$\frac{\partial\tilde{\phi}}{\partial t} = -g\eta - \frac{1}{2}\left|\frac{\partial\tilde{\phi}}{\partial x}\right|^2 + \frac{1}{2}\left(1 + \left|\frac{\partial\eta}{\partial x}\right|^2\right)\left(\frac{\partial\phi}{\partial z}\right)^2. \tag{3}$$