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Instability waves and aerodynamic noise in a subsonic transitional turbulent jet

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ABSTRACT

A subsonic transitional jet is simulated by large eddy simulation, while the flow and acoustic properties are validated against existed experimental results. In the hydrodynamic region, the total disturbance kinetic energy declines as azimuthal wavenumber (m) or frequency increases. In the far field, the acoustic field is dominated by the axisymmetric and first helical modes, and the peak of sound spectra at small polar angles to the jet axis are found be located in the range of Strouhal number $St = 0.2 - 0.3$. In order to understand the relationship between instability waves and noise generation, the parabolized stability equation (PSE) is solved for describing the evolution of instability waves. For the acoustic field, the linear model and nonlinear interaction model are built based on PSE solutions in combination with Lilley–Goldstein's analogy. The beam patterns of above two models are in agreement with experimental results qualitatively, while the nonlinear interaction model predicts more accurate directivity and locations of sound sources. Quantitatively, the linear model underestimates the sound pressure level (SPL) greatly. Nonlinear interaction can enhance the strength of sound sources and raise acoustic efficiency at small polar angles. In some cases, the nonlinear interaction of two instability waves can even increase SPL by nearly 20 dB. However, for $m \geq 2$, the nonlinear interaction model fails to predict reasonable results qualitatively.

1. Introduction

Extensive efforts have been devoted to study jet mixing noise and its generation mechanisms over decades, most of which is based on acoustic analogy approaches [1–3]. The understanding of 'sound sources' defined in acoustic analogy equations is limited and a universally agreed upon acoustic theory is still far off, especially for high Reynolds number jets, where turbulence plays a crucial role. Within the framework of Lighthill's acoustic analogy [1], the sources of jet mixing noise are corresponding to quadrupoles, which is unable to explain the characteristics like superdirectivity and the appearance of a broadband spectral peak at $St \approx 0.2$ at polar angles around 30° to the jet axis in the experiment of Stromberg, McLaughlin and Troutt (1980) [4]. After the large-scale coherent structures were found in turbulent shear flows [5], researchers soon realized the important role of these well-ordered structures in the process of noise generation [6]. The round jet is a typical flow that contains both small-scale turbulence structures and large-scale organized structures, and the motions of latter structures that are sometimes modeled by instability waves are believed to be responsible for dominant sound radiation concentrated at small polar angles [7] and thus have attracted a great deal of interest.

In supersonic jets or mixing layers, Tam and Burton (1984) [8] and Wu (2005) [9], and among others, have demonstrated a mechanism that the most unstable wave in the jet column is able to emit highly directional sound efficiently, named as Mach wave radiation, when its phase velocity relative to ambient speed of sound exceeds unity by a sufficient amount. This mechanism is usually interpreted in terms of the 'wavy-wall analogy' for clarity. In contrast to supersonic jets, the relationship between instability modes and noise generation is much more ambiguous in subsonic jets, because the instability modes propagate subsonically. At present, the idea that there exists some kind of connection between radiated noise and instability waves has been supported by experimental evidences [10]. However, the detailed role of instability waves in sound radiation remains an ongoing debate, and more concrete mechanism of generating suitable wavepacket is still required to be understood [11]. Based on classical linear hydrodynamic-instability theory, for a parallel flow, it is well known that the eigenfunction of a subsonic mode will decay to zero exponentially

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away from a jet or mixing layer, which means there would be no acoustic radiation produced by instability waves. Nevertheless, in real cases, nonlinearity plays a crucial role in flow development due to mean flow spreading. Under these combined effects, the instability wave will be firstly amplified, then saturated and finally attenuated, where the wave amplitude is varied in the flow direction. If we perform spatial Fourier transform of a flow variable (e.g. pressure), it is found that such spatial modulation would lead to a broadband wavenumber spectrum, a portion of which will be supersonic and radiated to far field as Mach waves, although the instability wave itself has a local phase speed that is subsonic relative to ambient stream, as demonstrated by Tam and Morris (1980) [12] and Tam (2009) [13]. Taking into account spatially developing instability waves in round jets, Sandham and Salgado (2008) [14] have proposed a simplified model for sound radiation from subsonic jets, in purpose of explaining the reason of the presence of a broadband spectral peak in experiments of Stromberg, McLaughlin and Troutt (1980) [4] and Viswanathan (2004) [15]. In their study, the linear parabolized stability equations (PSE) are solved to obtain the spatial developing instability modes of specified frequencies and wavenumbers based on a fixed base flow, the sound source terms in Lilley–Goldstein equation are computed using such instability modes by multiplication, and finally the sound radiation pattern could be obtained. To further clarify the linear and nonlinear mechanisms of sound radiation by instability waves, Suponitsky, Sandham and Morfey (2010) [16] performed direct numerical simulation of subsonic jets for a low Reynolds number of 3600. Their results suggest that nonlinear interaction of two instability modes (e.g. f_1 and f_2) is able to produce sound radiation efficiently at the difference frequency ($f_1 - f_2$) and the spectral peak observed in experiments can also be characterized by such a simple nonlinear interaction model. Different from Sandham et al. [14,16], using a matched-asymptotic-expansion approach, Wu and Huerre (2009) [17] studied the sound radiated by modulated wavepacket of helical modes, and proposed that the development of wavepacket is simultaneously influenced by non-parallelism and non-equilibrium effects. Their results indicate that the acoustic field is characterized by a single-lobed directivity pattern beam at an angle about 45°–60° to the jet axis, and a broadband spectrum centered at a Strouhal number $St \approx 0.07 - 0.2$.

In this study, we performed large eddy simulation for a subsonic transitional jet, which has a higher Reynolds number than Suponitsky et al. [16]. As we know, the flow development is inevitably changed significantly as the increase of Reynolds number [18], since the process of transition is faster, and the scales of vortical structures in the annular shear layer are finer. Multiple instability modes with relatively high frequencies are simultaneously imposed, then we can study the instability waves of non-direct forcing low frequency components in complicated flow fields. Using the time-averaged flow fields as the mean flow, taking into account non-parallel effect, the parabolized stability equations are solved for characterizing instability wave evolution. In order to identify the relationship between instability waves and sound, the linear and nonlinear interaction models are constructed based on PSE solutions and Lilley–Goldstein's analogy. The usage of LES for this study has some advantages. On the one hand, LES provides us with accurate acoustic field for specified frequency and wavenumber, which offers convenience for comparison. On the other hand, the mean flow given by LES is relatively accurate for characterizing instability waves. Nevertheless, it should be emphasized that the flow dynamics and acoustic features of transitional jets still be different from the experimental jets with fully turbulent nozzle-exit boundary layer conditions, which has been stated by Jordan and Colonius [11]. The importance of this issue has been recognized and investigated in recent experimental and numerical studies [19–22].

Using a posterior analysis based on LES mean flow, it enables us to get some understanding of the roles of instability wave in noise generation in the present transitional jet. In particular, the weakly nonlinear interaction model based on sound sources of Lilley–Goldstein's equation [14] is evaluated carefully, and we want to clarify to what extent such a model can be applied to relatively more complicated flow fields, by comparing with LES results. Based on this model, the objective of this study is to study qualitative and quantitative differences caused by nonlinear interactions of instability waves in a transitional subsonic turbulent jet. In addition, the present methodology can also be applied to even more complicated flow fields with fully turbulent inflow conditions.

The rest of this paper is organized as follows. Section 2 introduces the numerical approaches employed and detailed flow configuration, including the approach of large eddy simulation, solving procedures of parabolized stability equations and Lilley–Goldstein's equations, and etc. In Section 3, the major results are presented and detailed discussions are made. The flow and acoustic properties are presented and compared with existed experimental results. The flow development such as the evolution of the disturbance kinetic energy in the hydrodynamic region is investigated. Then, the linear model and nonlinear interaction model based on PSE solutions are constructed, and the model results are compared with that of LES qualitatively and quantitatively. Finally, some discussions and conclusions regarding the noise generation processes are summarized.

2. Numerical approaches and flow configuration

In this section, we briefly introduce the numerical approaches and flow configuration for computing near-field flow dynamics and far-field acoustic field.

2.1. Large eddy simulation and flow configuration

For flow development in the near field, we carried out large eddy simulation by solving the Favre-filtered compressible Navier–Stokes equations in cylindrical coordinates (r, θ, z), with the subgrid scales (SGS) modeled onto the resolved scales. The 7-point dispersion-relation-preserving (DRP) scheme [23] is utilized to compute radial and axial derivatives, while the Fourier spectral differentiation is adopted in the azimuthal direction. The low-storage version of the fourth-order Runge–Kutta scheme (RK) is used to advance the solution in time direction. Generally, in computation of the noise field, non-reflection boundary conditions introduced by Giles (1990) [24] is adopted to minimize spurious reflecting waves because of vortical structures passing through the boundary. In addition, buffer zones that enclose the entire physical domain are employed to damp the amplitude of strong waves. More details of solving techniques and procedures can be found in our previous work [25]. Presently, in order to get mesh-independent solutions, a total number of $N_z \times N_r \times N_\theta = 360 \times 165 \times 65$ grid points are used, which are distributed in the computational domain $z \in [-3r_0, 50r_0]$, $r \in [0, 21r_0]$, and $\theta \in [0, 2\pi]$, where r_0 is the jet radius. The physical domain is set up as $z \in [0, 38r_0]$, $r \in [0, 18r_0]$, and $\theta \in [0, 2\pi]$. Numerical results show that both near-field flow quantities and far-field sound are in good agreement with existed computations and experiments. In addition, in order to calculate converged statistics, two thousand flow fields with a time increment of $\Delta t = 0.1r_0/a_\infty$ are stored for posterior analysis. Kirchhoff's method [26] is employed for computing sound in the far field. For the first step, we store the pressure fields on the controlled surface located at $R_{KS} = 10r_0$ in numerical simulations. The propagation of pressure waves in