European Journal of Mechanics B/Fluids 57 (2016) 204–213

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

On the generalization of velocity slip in fluid flows using a steady-state series expansion of the wall shear stress: Case of simple Newtonian fluids

Martin Ndi Azese

Department of Mechanical Engineering, Texas Tech University, Lubbock, TX 79409, USA

ARTICLE INFO

Article history:
Received 8 January 2015
Received in revised form
23 December 2015
Accepted 12 January 2016
Available online 21 January 2016

Keywords:
Linear slip
Non-linear slip
Planar-Poiseuille
Planar-Couette
Newtonian fluid
Shear stress

ABSTRACT

This paper is one of two articles, where we present a new wall slip formulation based on a series expansion involving both differential in space and exponential forms of wall shear stress. In the first of these articles, we presented and described this new formulation using Phan-Thien–Tanner fluid as case study. Meanwhile, this second paper analyzes the new slip formulation for Newtonian Fluid. Unlike in the first paper, here, we have considered both the exponential and differential forms, though truncated at three terms each. Thus, we use our new truncated triple-slip-coefficients wall slip law to analyze Newtonian fluid in different systems. In particular, we study the planar Couette and planar Poiseuille problem, where two infinitely long and parallel plates have been used.

For this part also, slip velocities and shear stresses at the walls are scrutinized for both problems. Further, as the Couette problem is pressure independent; the differential form of the slip law is considered for Poiseuille problem only. In addition, the pressure and flow-rate are studied for various slip coefficients for Poiseuille flow case using condensed forms of the triple-slip-coefficients. Our results obtained prove reasonable as represented by physically realistic plots herein. This feasibility is especially dictated by the velocity profile across channel width for both application problems. More importantly, results obtained for Poiseuille problem is corroborated with experimental data. Therefore, we can infer from all these, that this new model has the potential of providing results which can match experimental data, that is, if the three slip-coefficients are properly chosen.

1. Introduction

In the past, researchers in the field of hydrodynamics were more interested in flow field inside a fluid domain. As a result, their attention was least focused on properly modeling of the boundary conditions at the periphery. Consequently, it was common to assume that the liquid layer closest to the solid boundary sticks to it meaning equal velocities for two interfaces in contact. This boundary condition is widely referred to as the "no-slip" boundary condition.

In 1827, this issue caught the attention of Navier [1], who constructed the first ever slip boundary condition. In his postulate, he suggested that the slip velocity be proportional to the tangential component of the stress, and this was referred to as the linear Navier-slip boundary condition. The fundamental definition of this initial postulate, [2,3], was based on the idea that for a velocity profile u the slip velocity of a liquid tangent to wall whose normal is $\mathbf{n}$ is defined as

$$\mathbf{u}_{slip} = \mathsf{L}_{\mathsf{slip}}[\mathbf{n} \cdot (\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathbf{T}})]_{\mathrm{t}}, \tag{1}$$

where $\mathbf{T}$ denotes transpose, $\mathbf{t}$ refers to the tangential-component, and constant parameter $\mathsf{L}_{\mathsf{slip}}$ has the dimension of length, thus the name slip length. We add that Maxwell who defined an identical slip velocity for cases of Newtonian gases also supported this law [4].

Typically, the slip length ($\mathsf{L}_{\mathsf{slip}}$) of common fluids have nanometer to micrometer order of magnitude [5,3]. Hence, when the characteristic length scale of flow system is not much larger than this slip-length, some variation of Eq. (1) should be taken into account to model slip. This suggests a non-dimensional number defined by the ratio of slip-length to a characteristic length scale dictated by the flow or the geometry (L_c). This deterministic slip ratio has been widely referred to as the slip-number. In fact, for gases the mean free path (l_m) is widely used in characterizing the degree of slip [4] whose ratio with characteristic distance defines the Knudsen number (Kn). Generally, $l_m \approx 500$ nm for liquids and according to this,

E-mail address: martin.azese@ttu.edu.

http://dx.doi.org/10.1016/j.euromechflu.2016.01.007

slip effects are important when the flow length-scale becomes of the order of a few micrometers.

Until recently, Navier-slip phenomenological law was slow to gain popularity as most researchers still continued to use the no-slip boundary condition [6,7]. Perhaps because the no-slip constraint seems to validate experimental predictions, contrary to viscous flows in closely confined conduits. Nonetheless, lately, fluid researchers have started considering these effects as some were noticing non-zero slip velocity of fluids on their bounding walls [8–14,5]. It is realized that a deeper understanding of this phenomenon will be especially crucial for microfluidic systems, [15], where slip length becomes around 10% of the relevant dimension of the domain. For these cases, underestimation of wall slippage can cause substantial error in calculation of velocity profile and axial evolution of flow.

The aforementioned systems are extremely relevant for contemporary science. For example, liquid encroachment through novel microcapillary channels [16–18] can be affected by the slip-velocity at the wall. Additionally, flow around conduit-bound microparticles [19–24] like blood cells should be analyzed by considering a proper slippage conditions at the surface of suspended bodies. However, such boundary conditions have never been taken into account in the flow analysis of mesoscale transport phenomena.

A proper estimation of this effect may have industrial impact, as it leads to significant corrections in formulations related to several modern technologies like, for example, micro-rheology [25] and polymeric micro-extrusion [26–28]. Remarkably, this new consideration can even be important in large-scale aerodynamic calculations for drag reduction and optimization if the boundary-layer thickness approaches to micron size.

With recent rapid advancements of measurement technique with high precision instruments, slip boundary conditions are gradually getting the attention of experimentalist. Their experiments have yielded direct or indirect evidence and measurement of slip. Such evidence have been brought to light by employing techniques such as particle image velocimetry (PIV) [15], near-field Laser velocimetry [29], streaming potential [30]. Further, measurements techniques have also included magnetic resonance imaging and laser Doppler anemometry [31], although we note that such imaging still suffers from limitation in spatial resolution.

In the present paper, our purpose is to continue the development of a semi phenomenological relation for wall slip velocity, which was first presented in our first paper, [32]. In this article, our new slip model is studied and assessed, considering Couette and Poiseuille planar flows of Newtonian fluids. Although existing slip laws provide a foundation in setting-up our formulation, we also intend to show that they can be considered as a sub-fold of our intended generalization of slip velocity. So, firstly, in Section 2 our new formulation together with a review of commonly used wall-slip equations are briefly presented. Next, we expect our new generalization to incorporate every fluid interaction at the neighborhood of a fluid–solid boundary of some common flow configurations. According to this, the new development is rigorously applied in a planar-Couette and planar-Poiseuille flow system in Section 3, together with their respective solutions. In the next segment, Section 4, we analyze and discuss the results and feasibility of the theory. Finally, this paper is summarized and concluded in Section 5.

2. Theory and formulation of wall slip

Since the postulation of the initial slip law by Navier, many alternatives have been formulated, [12], and the use of no-slip boundary condition has reduced considerably. Hence, in this section, we enumerate and concisely describe a few of the most common wall-slip theories. The choices of these slip laws, reviewed here, are also based on the fact that they include other sub-sets of familiar wall slip postulates. Thus, the review presented here, though aligned with the focus of this paper, is straight-to-the-point and could be inadequate for a complete understanding of slip phenomenon. For this reason, for additional familiarity with literature of wall slip we recommend that the reader visits the following three sources: First, the review paper that presents experimental studies of boundary slip by Neto et al. [12], and next, the book chapter Microfluidics: The no-slip Boundary Condition in [3], and finally, the review of slip (depletion) of polymer solutions in viscometers [27]. In addition, we also suggest the reading of review paper by Hatzikiriakos [33]—where rheology of both static and dynamic slip is reviewed in relation to polymer melts.

2.1. Partial review of existing approach

As mentioned before, an earlier attempt to address this slippage problem was put forward by Navier [1]. According to that theory (which is depicted in Eq. (1)), the slip velocity of a flow profile is proportional to the wall shear stress. It follows that this constant of proportionality is the ratio of slip distance b to viscosity η (i.e. b/η). This slip distance, pivotal to all subsequent models reviewed here, is a characteristic slippage length equivalent to the Navier slip distance $\mathsf{L_{slip}}$ in Eq. (1). It is defined as the stretch beyond the solid boundary up-to the position where the velocity must extrapolate to give zero velocity as defined in [3,34,35]. Moreover, as discussed in [3], the slip length has been found experimentally by different techniques, and some reported results show that it can be as small as 1 nm or as big as 2 μm. In particular, Haifeng et al. [15] used particle tracking technique to obtain sleep lengths within this range.

Since this inaugural postulation by Navier, many modifications and extensions have been brought to it. For instance, some experimental results [8] have suggested that slippage occurs at the walls only when the wall shear stress is greater than a certain critical value, τ_c. Hence for a wall perpendicular to $\hat{y}$, say, one of such extension of Navier slip follows,

$$\begin{aligned} U_{ws} &= \lambda[\tau_{xy} - \tau_c] \quad \text{if } \tau_{xy} \geq \tau_c, \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{2}$$

Here, τ_{xy} is again the wall shear stress, meanwhile τ_c is the critical stress, and λ is the slip coefficient. Kaoullas et al. [34] elaborated more on this connection.

Another extension of Navier slip is the non-linear Navier slip [10,36]. This slip law says that the wall slip velocity is proportional to the wall stress raised to a certain power. Because of that, it is also referred to as power-slip law. The general form of this slip law is

$$U_{ws} = k\left|\tau_{(wall,\ \delta)}\right|^m, \tag{3}$$

where m can be any real positive number, and δ is a characteristic slip distance describing the region of influence of slip within the fluid. Parameter δ, can also be defined as the fluid boundary thickness within which slippage is affected (i.e. distance within which slip effects are significant). Again, k is the slip coefficient, whose dimension is derived based on the value attributed to m.

Following the same concept of critical wall stress, Hatzikiriakos [8] developed a slip law with two coefficients, k_1 and k_2 in the form

$$\begin{aligned} U_{ws} &= k_1 \mathrm{Sinh}\{k_2|\tau_{xy}| - \tau_c\} \quad \text{if } \tau_{xy} \geq \tau_c, \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{4}$$