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Instabilities in unsteady boundary layers with reverse flow

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ABSTRACT

Instabilities arising in unsteady boundary layers with reverse flow have been investigated experimentally. Experiments are conducted in a piston driven unsteady water tunnel with a shallow angle diffuser placed in the test section. The ratio of temporal (Π_t) to spatial (Π_x) component of the pressure gradient can be varied by a controlled motion of the piston. In all the experiments, the piston velocity variation with time is trapezoidal consisting of three phases: constant acceleration from rest, constant velocity and constant deceleration to rest. The adverse pressure gradient (and reverse flow) are due to a combination of spatial deceleration of the free stream in the diffuser and temporal deceleration of the free stream caused by the piston deceleration. The instability is usually initiated with the formation of one or more vortices. The onset of reverse flow in the boundary layer, location and time of formation of the first vortex and the subsequent flow evolution are studied for various values of the ratio $\Pi_x/(\Pi_x + \Pi_t)$ for the bottom and the top walls. Instability is due to the inflectional velocity profiles of the unsteady boundary layer. The instability is localized and spreads to the other regions at later times. At higher Reynolds numbers growth rate of instability is higher and localized transition to turbulence is observed. Scalings have been proposed for initial vortex formation time and wavelength of the instability vortices. Initial vortex formation time scales with convective time, $\delta/\Delta U$, where δ is the boundary layer thickness and ΔU is the difference of maximum and minimum velocities in the boundary layer. Non-dimensional vortex formation time based on convective time scale for the bottom and the top walls are found to be 23 and 30 respectively. Wavelength of instability vortices scales with the time averaged boundary layer thickness.

1. Introduction

Our main concern in this work is the evolution of unsteady boundary layers with reverse flow. Unsteady flows occur in a variety of natural situations like pulsating flow in arteries, flow over flapping wings of birds and fins of fish, flow in the heart and some technologically important ones like on blade surfaces of compressor and turbine cascades in gas turbine engines. In the last case the periodically impinging wakes from the upstream blade row provide the unsteadiness.

For a 2-dimensional unsteady boundary layer (Fig. 1) the momentum equation in the flow direction is

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}. \tag{1}$$

Here x and y are the boundary layer coordinates, x along the surface and y normal to it; t is time; 'u' and 'v' are velocity components along x and y respectively. We will see that it is useful to decompose the pressure gradient term $\frac{1}{\rho}\frac{\partial P}{\partial x}$ into two components

$$\frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{\partial U_e}{\partial t} + U_e\frac{\partial U_e}{\partial x}, \tag{2}$$

where U_e is the velocity at the edge of the boundary layer. The first term of the right hand side of this equation may be called the temporal component ($\Pi_t = \frac{\partial U_e}{\partial t}$) which signifies acceleration or deceleration in time of the free stream and the second term is the spatial component ($\Pi_x = U_e\frac{\partial U_e}{\partial x}$) which represents the spatial or convective acceleration of the free stream. Many of the studies of instability in unsteady flows have been in straight tubes or channels, where the Π_x term is absent. However, in many cases, especially in biological systems both terms are present. An example is the flow over an unsteadily moving body of a fish or on the wings of a maneuvering aircraft. In this paper we are concerned with boundary

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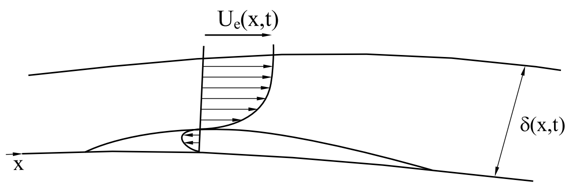

Fig. 1. Unsteady boundary layer.

layers with an adverse pressure gradient. The adverse pressure gradient can be due to Π_t or Π_x or both, i.e. deceleration of the outer stream can be in time or in space or in both.

The simplest and much studied class of problems is fully developed unidirectional flows, where only the Π_t component is present. Some classical unidirectional flow problems are a plane boundary impulsively moved in its own plane in a fluid at rest, flow due to a plate oscillating in its own plane, the starting flow or suddenly blocked flow in a pipe or channel [1] and oscillatory flows, with or without a mean component, in a constant cross-section duct. Analytical solutions may be obtained for laminar flow for two types of conditions. One is when the pressure gradient is given [2] and the other is when the volume flow rate with time is given [3,4]. Uchida [2] has given solutions for a sinusoidal variation of the pressure gradient in time.

Considerable research has been done over the years both on oscillatory flow (i.e. with zero mean) and the pulsating flow (i.e. with non-zero mean) over a flat plate and in pipes. In oscillatory flows the relevant thickness is the Stokes thickness $\delta_{st} = \sqrt{\frac{2\nu}{\omega}}$, where ω is the angular frequency of fluid oscillations and ν is the kinematic viscosity of the fluid. A relevant Reynolds number may be defined as $\mathrm{Re}_{st} = \frac{U_a \delta_{st}}{\nu}$, where U_a is the amplitude of the velocity variation. Another parameter is the Stokes parameter, $\Lambda = \frac{R}{\delta_{st}}$. R is the radius of the pipe. Experiments have shown that this type of flow may be classified into laminar, disturbed laminar, intermittently turbulent and fully developed turbulent flow [5]. The values of Re_{st} and Λ determine the type of flow. Hino et al. [5] have found the critical Reynolds number for turbulence $\mathrm{Re}_{st} = 550$ for $\Lambda > 1.6$.

Another class of unsteady unidirectional flows is when the flow is not periodic and unsteadiness is due to a single event, for example, flow in a pipe or channel that is suddenly blocked. Weinbaum and Parker [6] have investigated the laminar decay of a suddenly blocked channel and pipe flows. They have obtained the velocity profile using a Pohlhausen type technique. Later Das and Arakeri [4] have proposed a procedure to obtain analytical solutions for an unsteady unidirectional laminar flow in a pipe with circular cross section or in a two dimensional channel created by an arbitrary but given volume flow rate variation with time. Ghidaoui and Kolyshkin [7] have proposed a matched asymptotic expansion method for flows where the acceleration/deceleration is very rapid, and the diffusion of momentum near the wall is confined to a small fraction of the pipe diameter.

In both types discussed above the velocity profiles develop inflection points when the pressure gradient is adverse and often leads to instability at relatively low Reynolds numbers. This instability may result in vortex formation and subsequent breakdown of the vortices makes the flow turbulent. The non-dimensional time scale of the appearance of waves, formation of vortices and transition were reported for the flow in a pipe [8] where the mean velocity had a trapezoidal variation with time. From the linear stability analysis they have shown that wavelength of the vortices and time to vortex formation scale respectively with boundary layer thickness (δ) and the convective time scale $\frac{\delta}{\Delta U}$. Fig. 2 shows some relevant parameters for the instability of a boundary layer with reverse flow. The distance of the inflection point from the wall affects the value of the critical Reynolds number [3]. For $\mathrm{Re} > \mathrm{Re}_{cr}$, the instability is inviscid and growth rates of perturbations are essentially determined by δ and ΔU.

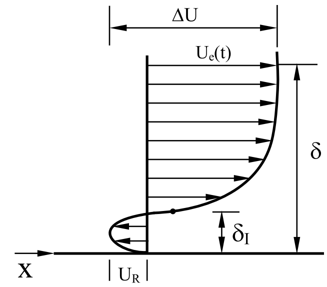

Fig. 2. Velocity profile in a straight channel showing the boundary layer parameters. U_e—velocity at the edge of the boundary layer, U_R—reverse flow magnitude, δ_I—distance of the inflection point from the wall and $\Delta U = U_e - U_R$.

An adverse pressure gradient can also cause boundary layer separation. Boundary layer separation can be viewed as 'breaking away' of the flow from the surface and consequent breakdown of the boundary layer assumptions. Under steady conditions the separation point can be considered as the point where the wall shear stress vanishes, $\tau_w = 0$ [1], the Prandtl condition. A Goldstein singularity occurs in the solution of the steady boundary layer equations at such points. If the boundary layer reattaches, a separation bubble is obtained. The separation bubble could be thin and the boundary layer assumption still be valid. Depending on the length, a bubble can be classified as short or long. If the bubble length is of the order $10^2\delta_s^*$ to $10^3\delta_s^*$, where δ_s^* is the displacement thickness at separation, the bubble is called short and it is considered long if the length is of order $10^4\delta_s^*$ [9]. Gaster [10] has studied experimentally the behavior of a laminar separation bubble on a flat plate. Under certain conditions he has seen the bursting of the separation bubble and shedding of vortices. Later, several numerical studies have revealed the structure of a 2-D laminar separation bubble [11,12] and there is a quite good agreement with the experimental results. Pauley et al. [11] have found that weak adverse pressure gradient results in the formation of steady separation bubble, but when the pressure gradient is strong enough it leads to the formation of separated shear layer and shedding of vortices downstream of the reattachment point. Direct numerical simulations [13,14] give some insight into the instability mechanism in short laminar separation bubbles. Stability of laminar separation bubble formed on a flat plate under imposed pressure gradient has been studied in detail [15].

Under imposed unsteady conditions, boundary layer separation is not yet fully understood. There is no specific criterion for its occurrence, in contrast to steady separation. For unsteady flows, the point of zero shear stress need not correspond to the point of separation. A thin reverse flow region can occur in which the boundary layer approximation is still valid. The thickening of this thin reverse flow region may result boundary layer separation. A separation criterion proposed by Moore [16], Rott [17] and Sears and Telionis [18] (known as MRS criterion) says that unsteady separation occurs at zero shear stress location and velocity of the separation point equals to the local velocity in the streamwise direction. Zero shear stress here need not occur at the wall. For steady cases this criterion reduces to Prandtl's criterion. Van Dommelen and Shen [19] have performed a numerical study of the unsteady separation in the Lagrangian frame. Using a dynamical system approach Haller [20] has extended Prandtl's separation criteria to two dimensional unsteady flows with no slip boundary condition at the wall. He has defined unsteady separation as a material instability induced by an unstable manifold of a distinguished boundary point.

Many studies of unsteady boundary layer separation have been in relation to dynamic stall. Unsteady boundary layer separation