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An analytical study on hydrodynamics of an unsteady flow and mass transfer through a channel asymmetrically lined with deformable porous layer

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HIGHLIGHTS

- Endothelial-cell glycocalyx layer(EGL) influences the blood plasma flow rate.
- Hydrodynamics of flow through EGL is modeled with biphasic mixture theory.
- Rate of solute transfer from plasma to EGL is calculated at the interface.
- Higher depth of EGL causes reduction of flow rate through free lumen.
- Permeation velocity controls the solute transport in the endothelial cells.

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ABSTRACT

The present study is motivated by its possible relevance to the rheology of blood and the transport of nutrient and drug molecules (macro–microscopic level) through capillaries, venules and digestive tract under the influence of endothelial-cell glycocalyx (EG). As an approximation, we study unsteady fluid flow through a rectangular channel lined with asymmetric porous lining. In order to model a more realistic situation, we adopt biphasic mixture theory for the unsteady hydrodynamics of fluid transport in poroelastic layer(s) (EG). The flow within the lumen region between the deformable porous layers is governed by unsteady Stoke's equation. The unsteady nature of the flow is being taken care with the Fourier series expansion analysis. Shear stress is calculated at the interface of free lumen and porous layers along with volumetric flow rate through the free lumen region. An expression for Sherwood number is derived at the said interface from the pseudo steady state solution of the mass transfer equation that quantifies the transport of solute nutrient through the lumenal space. The corresponding convection–diffusion equation is solved within the mass transfer boundary layer using similarity transformation method. It is observed that the rate of volumetric flow through the free lumen increases and interfacial shear stress (i.e., shear stress at the interface of at the porous and free lumen) decreases with stress jump coefficient at the mentioned interface. Length averaged Sherwood number deceases with increase in depth of the porous layer and increases with the constant permeation velocity when the other parameters characterizing the mass transfer process remain constant.

1. Introduction

Endothelial-cell glycocalyx (EG) covers the surface of the endothelial cells luminally and it consists of oligosaccharide like carbohydrate. It covalently binds with a network of membrane bound proteoglycans and glycoprotein. EG serves as a vascular permeability barrier which shields the vascular wall from the direct exposure to blood flow within the vascular endothelium. While the protective functionality of EG is universal, its relative importance varies depending on the location it resides within vasculature. Over the past few decades, the role of glycocalyx in vascular physiology and pathology along with hemostasis, interaction between blood cell and vessel wall etc., has gained enough interest in the field of Biomechanics and transport processes inside physiological systems [1]. Glycocalyx has a significant role in controlling delivery of nutrients and hormones to the tissues and it may be vital in coordinating the enhanced blood flow rate. It can be noted that glycocalyx influences some adverse physiological conditions like

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hypo and hyper glycemia, ischemia, atherosclerosis, arteriosclerosis etc. From the hydrodynamics point of view, this layer can be regarded as a hydrated gel with small portion (normally, 1%–2%) of fibrous tissue [2]. Plasma proteins like albumin, fibrinogen etc. can be absorbed within glycocalyx and thus these become a part of the fibrous matrix of macromolecules that include the glycocalyx. Within capillaries and small blood vessels, glycocalyx layers may have a thickness up to 4 μm [3].

Hydrodynamics of unsteady flow between two symmetrical glycocalyx layers has been studied by Barry et al. [4]. There, the glycocalyx layer has been considered as deformable porous layers (DPL) and unsteady behavior has been studied via Fourier series expansion techniques. The unsteady flow induced infinitesimal deformation of porous layer has been discussed by Barry and Aldis [5]. Perturbation technique has been used to obtain an approximate solution for the constitutive equations which are non-linear due to the non-linear nature of the permeability. It is customary to model glycocalyx, as a hydrodynamic problem of flow inside a channel with porous lining [4] or movement of rigid pellet in a cylindrical tube lined with deformable porous layer [3]. The latter study has a close relevance with rheology of blood in the microcirculation. According to Damiano et al. [3], a feeble shear stress can be induced by glycocalyx at the endothelial-cell membrane. This phenomena plays a significant role in the mechano-transduction of flow which is a part of an active metabolic process like autoregulation. The physiological flows in capillaries, venules, and the pleural space has been modeled by Wei et al. [2], considering pressure driven Newtonian flow in two dimensional wavy walled channel lined with poroelastic layer. Damiano and Stace [6] provided an appropriate relationship between the mechano-electrochemical properties of glycocalyx and physiologically realistic constitutive models that are developed under the assumption of unidirectional flow. In all of the above studies, the glycocalyx is modeled as a thin poroelastic layer lining the capillary wall. The said layer is assumed to be a mixture of viscous fluid constituent (linear) and isotropic, highly compressible elastic solid constituent (linear) with a small solid-volume fraction. In order to model the corresponding situation mathematically, the concept of biphasic mixture theory [4,3,7] can be used. Some of the recent studies by Liu and Yang [8], Shaw et al. [9] dealt with the issue of electrokinetic effect on electrolyte flow within free lumen region between endothelial-cell glycocalyx layer (EGL). Hydrodynamics of such flow mechanism is mainly of two phases, where the EGL solely contains a Newtonian fluid and blood flow in between the layers is treated as non-Newtonian Casson fluid. Though there are studies on understanding various phenomenon inside EGL, there is a need to understand better. This is due to the fact that mall function of many organs in a human body can be related to physical and chemical changes taking place in EGL residing inside these organs. Hence, any attempt to understand the hydrodynamic phenomena coupled with mass transfer related to EG helps one to know better, the biological behavior.

Kudo et al. [10] have indicated that the uptake of albumin into an endothelial cell is necessarily an energy dependent process, where at low shear stress ATP (Adenosine triphosphate) synthesis increases with albumin uptake and vice versa. Study of Ueda et al. [11] finds that compared to zero shear stress condition, albumin uptake into endothelial cells is increased by 16% at low shear stress condition. On the other hand, at high shear stressed condition albumin uptake is decreased by 30%. According to Shaaban and Duerinckx [12], low and oscillating shear stress has a potential impact on the development of atherosclerosis as determined by the inverse relation between wall shear stress and arterial wall thickness. Their study also provides some insight about the effect of some known risk factors on atherosclerosis and how changes in the risk factors can affect wall shear stress. Sun et al. [13] have investigated the influence of wall shear stress and transmural pressure on the low density lipoprotein (LDL) accumulation in the artery wall which is a multilayered porous media. According to this investigation, low wall shear stress is mainly responsible for LDL accumulation by weakening the process of convective clearance in transmural flow. At low shear stress range (<1 Pa), albumin uptake by artery wall increases with increasing wall shear stress. But, for shear stress range (>2 Pa), albumin uptake by artery wall decreases with increasing wall shear stress. The process of albumin uptake is so slow that long time is required for albumin to reach a critical value. Consequently, this process can be considered to be steady.

Glycocalyx provides additional surface for adsorption of nutrients and drug molecules from the gastrointestinal tract (specially in the small intestine) where it can be found on the apical portion of microvilli. Endothelial-cell glycocalyx layer (EGL) within blood vessel and gastrointestinal tract is semipermeable towards the anionic molecules such as albumin, globulin, fibrinogen and other plasma proteins, which are able to penetrate the EGL when they possess a specific structure and size [14]. In this connection, we would like to proceed for a mathematical modeling of vascular solute nutrients and plasma protein transport where EGL is responsible for absorption of solute molecules that are being convected and diffused through the blood like body fluid. This motivate us to go through some related literature where transport reaction mechanism of chemical reactive fluid is studied within rectangular and cylindrical geometries with the walls coated with porous layers. Essentially, the porous layers are taking part in the process of absorption. Ng and Rudraiah [15] have discussed the process of mass transport accomplished by reaction, convection, and dispersion, of a chemically active solute in steady Poiseuille flow through a tube whose wall is coated with a reactive substance. A detailed mass transfer modeling of a catalytic reaction system in a micro-reactor with the walls coated with catalyst has been reported in the work of Sharma et al. [16]. Mass transport process of an electrically neutral solute dissolved in an electrolyte solution flowing through a microtube in presence of external electric field has been studied by Vennela et al. [17]. A Sherwood number correlation has been proposed in order to quantify the permeation flux through the porous wall of a microtube. In another study, Vennela et al. [18] have repeated the above study in a porous rectangular microchannel and proposed a correlation with Sherwood number in order to quantify the mass transport through the permeable wall. In the above two studies, the convection–diffusion equation for the transport of dissolved solute has been investigated within the mass transfer boundary layer with the help of similarity transformation method. In a recent study, Mondal and De [19] have reported a correlation with Sherwood number in order to quantify mass transfer process of neutral solute dissolved in an electrolyte which obey power law, when solutes move into the porous wall in presence of permeation flux at the wall.

Present article deals with the situation where solute nutrients, LDL and plasma proteins like albumin, fibrinogen etc. that are carried through the plasma of blood in a microvessel. The endothelial-cell glycocalyx layer adjacent to the wall of microvessel does not have uniform thickness at all the places. It is assumed that the curvature effect can be neglected due to the small diameter of the microvessel. Consequently, the same can be approximated as a channel with EGL lined in the adjacent microvascular wall. It is assumed that the said layer is poroelastic and asymmetrically lined with the wall of microvessel. This assumption takes care of the fact that the thickness of EGL is non-uniform. In what follows, we call the glycocalyx layers as poroelastic layers Ω_{p_I} and $\Omega_{p_{II}}$. Blood is assumed to flow through the lumen region in between Ω_{p_I} and $\Omega_{p_{II}}$. We assume that