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An embedded boundary approach for the simulation of a flexible flapping wing at different density ratio

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ABSTRACT

We have developed a strong-coupling approach based on a uniformly-applied Eulerian description for both fluid and solid and provided a simple monolithic formulation to compute highly flexible structures interacting with surrounding fluid flows. Using a fast-tracking method and a fast solver for the modified pressure equation with variable density, we keep the same low computational cost as in the uniform density case studied previously. The new algorithm is first validated by the simulation of the self-sustained oscillation of a flexible plate. Then, it is applied to study the effect from density ratio on a flexible plate flapping with incoming flow. The simulation shows strong effect of density ratio on the pattern of fluid–structure interaction and the propulsion performance through the change in mass ratio and frequency ratio.

1. Introduction

Birds and insects have survived by taking the advantage of flapping flexible wings for high energy efficiency and incredible maneuverability, when fixed rigid wings conventionally used for airplanes fail to meet the need at the same low Reynolds number. The flapping-wing mechanism from the nature has inspired generations of aerial vehicle designs from ancient flight machines to modern unmanned aerial vehicles. Especially, with the recent demand of micro air vehicles (MAVs), flapping-wing design attracts attention by many desirable characteristics (i.e. efficiency, maneuverability, hovering-capability) at low Reynolds number regime [1].

Starting with the pioneer work by Knoller [2] and Betz [3] in thrust generation by a plunging airfoil, numerous research in experiments and numerical simulation has been done to understand the propulsion by plunging and pitching foils and has been summarized in various places [4–7]. Because of extra complexity brought in by wing flexibility, the majority of earlier works focused only on rigid wings [4,8–10] or prescribed deformable wings [11,12]. However, recently, there were increasing interests and number of works in truly flexible wings with fully coupled fluid–structure interaction in both experiments and numerical simulation [7,13,14].

To simulation fluid–structure interaction, the fluid and solid are typically solved by separate equations and algorithms, and they are then coupled at the interface through boundary conditions. There have been many studies for different applications and using different numerical algorithms. Donea et al. [15] applied Arbitrary Lagrangian–Eulerian (ALE) finite element method to solve the fluid and study fluid–structure systems under transient dynamic loading. To understand the flexibility effect of a flapping foil, Zhu [16] used boundary element method to solve the fluid and coupled it with a two-dimensional thin-plate structural model by iterations. Luo et al. [17], in their study of the vocal fold vibration in human phonation, used immersed boundary method to solve the incompressible fluid equation with moving boundary and coupled with a linear viscoelastic solid equation for interaction. Tian et al. [18] applied a similar approach to study the aerodynamics of elastic insect wings. All the above approaches require accurate and explicit representation of boundary conditions (e.g. location, velocity, force) at the fluid–solid interface which link the fluid and solid solvers and play the key role in the convergency between these two solvers. In fact, the convergency is not guaranteed in some cases, and some works resorted to choose weak coupling (i.e. without any interaction) to avoid the convergency problem and save computational time.

In our study, we used a strong-coupling approach to simulate a highly flexible wing interacting with surrounding fluid flow in a globally Eulerian framework for both fluid and solid, which avoids entirely the explicit representation and matching of boundary conditions at fluid–solid interface and the associate problem in

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the convergency of two separate solvers. The idea has been based on an immersed boundary approach solving a combined Eulerian fluid–solid equation proposed independently by Zhao et al. [19] (for convenience, denoted later as ZFM) and by Boffi et al. [20]. ZFM extended the standard immersed boundary method by Peskin [21] to work on both membrane and non-membrane structures. The force-projection method developed in ZFM is to treat the solid body force and surface force in a combined manner despite the difference in singularity properties, which distinguishes itself from the more traditional approach adopted by Griffith and Luo [22]. To control the flapping trajectory, Yang et al. [14] later modified ZFM algorithm to include control cells (denoted as YWZ). In both ZFM and YWZ formulations, they made a strong assumption of applying the same density for both solid and fluid. This assumption largely simplified the derivation, however, it also made impossible to study the effect from density ratio which was pointed out by Ishihara et al. [13] as one of the basic similarity parameters in flapping flight. In this paper, based on YWZ formulation, we further extend the algorithm to handle arbitrary density ratio. A cell-tracking method and an interface density smoothing approach are introduced to dynamically define the density field on Cartesian mesh. Then, the variation of density is included by modifying the original pressure Poisson equation with a variable density ratio coefficient and the momentum equation with variable density for different domain regions. The new approach allows us to study the propulsion features and structure responses by flexible wings with different densities.

For the rest of the paper, the detail algorithm is described in Section 2, the numerical simulation results and discussion are in Section 3, and the final conclusion is presented in Section 4.

2. Numerical algorithm

For the completeness of the paper, some key ideas from ZFM and YWZ are briefly revisited and included in this session. (For clarity, we use "$*$" to denote dimensional variables in this paper; the variables without "$*$" are non-dimensionalized accordingly by the chord length L^*, incoming velocity U^*, fluid density ρ_f^*, and fluid viscosity μ_f^*.)

2.1. Governing equations

Compared to the Navier–Stokes equation for incompressible flow,

$$\begin{aligned} \nabla \cdot \mathbf{u} &= 0, \\ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} &= -\nabla p + \frac{1}{Re_f} \nabla^2 \mathbf{u}, \end{aligned} \tag{1}$$

where the Reynolds number for fluid is $Re_f = \rho_f^* U^* L^* / \mu_f^*$, the momentum equation for viscoelastic solid typically has a different form:

$$\rho_s \frac{D\mathbf{u}}{Dt} = \nabla \cdot \sigma_v + \nabla \cdot \sigma_e, \tag{2}$$

where ρ_s is solid density, σ_v is viscous stress tensor and σ_e is elastic stress tensor. When the solid is considered incompressible, σ_e can be decoupled in a way similar to the derivation of an incompressible fluid:

$$\sigma_e = -p\mathbf{I} + \boldsymbol{\tau}_{elas}, \tag{3}$$

where p is the pressure to enforce the incompressibility and $\boldsymbol{\tau}_{elas}$ is the deviatoric elastic stress tensor. When a simple nonlinear neo-Hookean model is considered, we have

$$\boldsymbol{\tau}_{elas} = \mu_s (\mathbf{A} \cdot \mathbf{A}^T - \mathbf{I}), \tag{4}$$

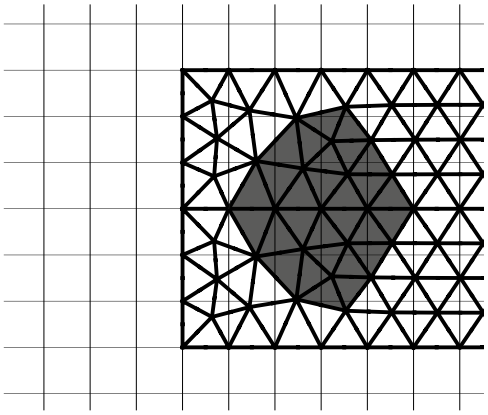

Fig. 1. The sketch of mesh configuration for a solid plate in fluid flow: Cartesian mesh with thin lines is the global Eulerian mesh for both fluid and solid; triangles with thick lines are the local Lagrangian mesh for solid (Ω_s); dark triangles are solid control cells (Ω_c).

where the solid shear modulus $\mu_s = \mu_s^* / \rho_f^* (U^*)^2$. When the same density and viscosity are assumed for solid and fluid (as in ZFM [19]), the above solid momentum equation (2) can then be formulated in Eulerian framework in a form resembling Navier–Stokes equation for fluid:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re_f} \nabla^2 \mathbf{u} + \nabla \cdot \boldsymbol{\tau}_{elas}. \tag{5}$$

At the fluid–solid interface, no-slip condition and force balance are enforced by:

$$\begin{aligned} [\mathbf{u}]|_\Gamma &= 0, \\ [(-p\mathbf{I} + \mu_f (\nabla \mathbf{u} + \nabla \mathbf{u}^T)) \cdot \mathbf{n}]|_\Gamma &= \boldsymbol{\tau}_{elas} \cdot \mathbf{n}, \end{aligned} \tag{6}$$

where $[q]|_\Gamma$ denotes the jump across the interface Γ, and $\mathbf{n}$ is the unit normal vector [19].

With the similarity between (1) and (5), the fluid and solid equations along with the boundary condition at the interface (6) can be combined to one single equation for both fluid and solid,

$$\begin{aligned} \nabla \cdot \mathbf{u} &= 0, \\ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} &= -\nabla p + \frac{1}{Re_f} \nabla^2 \mathbf{u} + \nabla \cdot (\chi_s \boldsymbol{\tau}_{elas}), \end{aligned} \tag{7}$$

where the characteristic function χ_s is defined by

$$\chi_s = \begin{cases} 1 & \text{in } \Omega_s \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

to embed elasticity only in solid area Ω_s (Fig. 1). The solid force term $\nabla \cdot (\chi_s \boldsymbol{\tau}_{elas})$ is equivalent to the combination of the body force $\mathbf{B}$ on Ω_s and the surface force $\mathbf{F}$ on Γ:

$$\begin{aligned} \mathbf{B} &= \chi_s \nabla \cdot \boldsymbol{\tau}_{elas}, \\ \mathbf{F} &= -\int_\Gamma \delta(\mathbf{x} - \mathbf{x}_\Gamma(\varepsilon, t)) \boldsymbol{\tau}_{elas} \cdot \mathbf{n} d\varepsilon, \end{aligned} \tag{9}$$

where $\delta(\mathbf{x})$ is Dirac delta. Zhao [23] showed that the singular surface force $\mathbf{F}$ enforces the force balance in (6) at the interface. It is worth noting that the hydrodynamic force on solid boundary does not appear explicitly in fluid–solid equation (7) since it becomes an internal force in the combined formulation. By avoiding the computation for values explicitly at the interface, the unified formulation for solid and fluid reduces the numerical errors and the risk of instability, and the monolithic formulation largely increases the computational efficiency as well.

However, the above formulation lacks the mechanism to define the motion of certain components for desirable moving trajectories