European Journal of Mechanics B/Fluids 55 (2016) 162–169

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Errors due to a practical Green function for steady ship waves

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ARTICLE INFO

Article history:
Received 31 May 2015
Received in revised form
3 August 2015
Accepted 25 September 2015
Available online 8 October 2015

Keywords:
Ship waves
NEUMANN–MICHELL theory
Green function
Analytical approximation

ABSTRACT

Errors that stem from a practical analytical approximation L_a to the local flow component L in the Green function associated with steady linear potential flow around a ship hull are considered. Although the approximation L_a is not very accurate, the flow potentials evaluated via the exact local flow component L or the approximation L_a for Froude numbers $F = 0.15$, 0.3 and 0.5 cannot be distinguished, except at $F = 0.15$ for which relatively small differences can be observed. Moreover, the sinkage, the trim angle and the wave drag predicted by the Neumann–Michell (NM) theory, with the local flow potential evaluated using L or L_a, are in very close agreement. Despite its remarkably simplicity, the analytical approximation L_a can then be used to compute the local flow component L in the Green function, in the entire flow region, within the framework of the NM linear potential flow theory and the related Hogner approximation. The practical analytical approximation L_a is far more practical than the basic integral representation of L, and is an important element of the NM theory. Indeed, the analytical approximation L_a, and other features of the NM theory, make it possible to compute the flow around a steadily advancing ship hull in a highly efficient way, as required for routine practical applications to design and hull-form optimization.

1. Introduction

Most ships are streamlined slender bodies that operate at high Reynolds numbers $10^9 < R_e$. Viscous effects are then confined within very thin boundary layers, and potential-flow theory provides a realistic framework for computing the flow around a ship hull that advances at constant speed along a straight path in calm water. Indeed, potential-flow theory is adequate, as well as practical and useful, for applications to ship design (notably early design) and especially hull-form optimization. Potential flow around a (steadily advancing) ship hull is then considered here.

In particular, the Neumann–Michell (NM) theory and the related Hogner approximation considered in [1–3] are of main interest here. This practical theory is shown in [2–7] to yield predictions of the sinkage, trim and drag experienced by a ship, as well as predictions of the wave profile along a ship hull and of far-field waves along a longitudinal cut behind a ship, that are in satisfactory agreement with experimental measurements for a range of ship models and Froude numbers. The NM theory is then useful for ship design, and is especially well suited for hull-form optimization as amply demonstrated in [8–16].

The NM theory is based on a Green function that satisfies the radiation condition and the classical Kelvin–Michell linearized boundary condition at the free surface. This Green function is then a major element of the NM theory, and of the related Neumann–Kelvin (NK) theory given in [17,18] and further considered in a broad literature, reviewed in [2]. The Green function used in the NK and NM theories is considered in an extensive literature, reviewed in [2,19]. The Green function $G(\tilde{\mathbf{x}}, \mathbf{x})$ is given by the sum of three basic components: (i) the fundamental free-space Green function $-1/r$ where r denotes the distance between the source point $\mathbf{x} \equiv (x, y, z)$ and the field point $\tilde{\mathbf{x}} \equiv (\tilde{x}, \tilde{y}, \tilde{z})$ in the Green function, (ii) a wave component G^W that consists of a linear superposition of elementary plane waves and is given by a single Fourier integral with continuous integrand, and (iii) a non-oscillatory local flow component G^L that is defined by a double Fourier integral with singular integrand. This impractical singular double Fourier integral can be transformed into a single integral with integrand expressed in terms of the complex exponential integral function $E_1(\cdot)$.

The basic decomposition of the Green function G into a wave component G^W and a local flow component $-1/r + G^L$ is not unique. Three alternative decompositions are given in [20], and

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http://dx.doi.org/10.1016/j.euromechflu.2015.09.009
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the decomposition recommended in that study is used here. Complementary near-field and far-field asymptotic approximations of the local flow component are given in [21–24]. Practical numerical approximations based on Chebyshev polynomials [24] or table interpolation [25–27] in complementary contiguous regions have also been given. Simple analytical approximations to the local flow component G^L are given for the two special cases $\tilde{y}-y=0$ or $\tilde{z}+z=0$ that correspond to flows due to thin ships [28] or to air-cushion-vehicles or planing boats [29]. These two special analytical approximations are extended in [19] to the general case

$$-\infty<\tilde{x}-x<\infty \quad -\infty<\tilde{y}-y<\infty \quad -\infty<\tilde{z}+z\le 0. \tag{1}$$

The analytical approximation to the local flow component G^L given in [19] is valid within the entire lower half space (1) as just noted, and is shown in [19] to provide a practical way of evaluating the flow created by a distribution of sources over an arbitrary ship hull surface. In particular, the method considered in [19] to numerically evaluate the flow due to a surface distribution of sources only involves elementary continuous functions (algebraic, exponential and trigonometric) of real arguments. This method is used in [2–7]. The satisfactory overall agreement between numerical predictions and experimental measurements reported in these studies, as well as the comparison between the local flow component G^L evaluated via the integral representation of G^L or the related analytical approximation G^L_a given in [19], suggest that the simple approximation G^L_a is sufficiently accurate for most practical applications. Nevertheless, a more precise estimate of the errors due to the analytical approximation G^L_a is highly desirable. This study of numerical errors due to the approximation $G^L\approx G^L_a$ is considered here.

2. The Green function

Potential flow around a ship hull of length L_s that advances at constant speed V_s along a straight path in calm water of effectively infinite depth and lateral extent is considered. The flow is observed from a moving system of Cartesian coordinates $\mathbf{X}\equiv(X,Y,Z)$ attached to the ship and thus appears steady. The X axis is chosen along the path of the ship and points toward the ship bow. The Z axis is vertical and points upward, with the undisturbed free surface taken as the plane $Z=0$. The flow velocity in this system of coordinates is $(U-V_s,V,W)$ where $\mathbf{U}\equiv(U,V,W)$ denotes the disturbance flow velocity due to the ship. A reference length L_{ref}, commonly chosen as the ship length L_s or as V_s^2/g where g is the acceleration of gravity, is used to define nondimensional coordinates

$$\mathbf{x}\equiv(x,y,z)\equiv(X,Y,Z)/L_{ref}. \tag{2}$$

The Froude number F is defined as

$$F=V_s/\sqrt{gL_{ref}}. \tag{3}$$

The choice $L_{ref}=V_s^2/g$ yields $F=1$.

The classical theoretical framework associated with a Green function $G(\tilde{\mathbf{x}},\mathbf{x})$ that satisfies the usual Kelvin–Michell linearized free-surface boundary condition for steady flows and the radiation condition is adopted. This Green function represents the velocity potential of the flow created at a flow-field point $\tilde{\mathbf{x}}\equiv(\tilde{x},\tilde{y},\tilde{z}\le 0)$ by a unit source at a point $\mathbf{x}\equiv(x,y,z\le 0)$, as well known. The (nondimensional) distances between the flow-field point $\tilde{\mathbf{x}}$ and the source point $\mathbf{x}$ or its mirror image $\mathbf{x}_1\equiv(x,y,-z)$ with respect to the mean free-surface plane $z=0$ are given by

$$r\equiv\sqrt{(\tilde{x}-x)^2+(\tilde{y}-y)^2+(\tilde{z}-z)^2} \tag{4a}$$

$$r_1\equiv\sqrt{(\tilde{x}-x)^2+(\tilde{y}-y)^2+(\tilde{z}+z)^2}. \tag{4b}$$

The nondimensional coordinates (a,b,c) and the related distance d are defined as

$$a\equiv\frac{|\tilde{x}-x|}{F^2}\qquad b\equiv\frac{|\tilde{y}-y|}{F^2}\qquad c\equiv\frac{-(\tilde{z}+z)}{F^2} \tag{5a}$$

$$d\equiv\sqrt{a^2+b^2+c^2}\equiv r_1/F^2. \tag{5b}$$

One has $0\le a$, $0\le b$, $0\le c$ and $0\le d$.

The Green function G can be expressed as

$$4\pi G=-1/r+G^L+G^W \tag{6}$$

where $-1/r$ is the fundamental free-space Green function, and G^L and G^W represent a local flow component and a wave component that account for free-surface effects. The basic decomposition (6) into waves and a local flow is not unique, as already noted. The decomposition recommended in [20] is adopted.

The wave component G^W in this decomposition is given by the Fourier superposition of elementary waves

$$G^W=H(x-\tilde{x})\frac{4}{F^2}\,\mathrm{Im}\int_{-k_\infty}^{k_\infty}dk\,\Lambda\,E\quad\text{with}$$

$$E\equiv e^{(1+k^2)(\tilde{z}+z)/F^2+i\sqrt{1+k^2}\,[\tilde{x}-x+k\,(\tilde{y}-y)]/F^2} \tag{7}$$

where $H(\cdot)$ is the usual Heaviside unit-step function, and Im means imaginary part. Moreover, the finite limits of integration $\pm k_\infty$ and the 'filter function' Λ remove unrealistic short waves that correspond to $k_\infty<|k|$, as is necessary for practical purposes [30,31]. The contribution of the wave component G^W to the flow potential can be evaluated in a practical manner via the classical Fourier–Kochin approach, as considered in [2,3], and this approach is used here. The local flow component G^L in expression (6) for the Green function is now considered.

3. Local flow in the Green function

The local flow component G^L in (6) is expressed in [20] as

$$G^L=1/r_1-2\,L/F^2. \tag{8}$$

Here, L is defined by the basic integral representation

$$L\equiv 1+\psi-\frac{1}{\pi}\int_{-1}^{1}dt\,\mathrm{Im}[e^M E_1(M)+\ln(M)]$$

$$\text{with } M\equiv(bt-c\sqrt{1-t^2}+ia)\sqrt{1-t^2}. \tag{9}$$

Moreover, $E_1(\cdot)$ stands for the complex exponential integral function, and ψ is defined as

$$\psi\equiv c/(d+a). \tag{10}$$

Numerical evaluation of the integral (9) is overly time consuming for practical applications.

The more practical approach based on the simple analytical approximation

$$L\approx L_a\equiv 1/(1+d)+\psi/(1+d)^2+\frac{0.2\,d}{(1+d)^5}\left[\left(A+\frac{B\,c}{\sqrt{b^2+c^2}}\right)\left(1-\frac{a}{d}\right)-\frac{Ca}{d}\right] \tag{11a}$$

with

$$A\equiv 4+6\,d+26\,d^2 \tag{11b}$$

$$B\equiv 1+39\,d-24\,d^2 \tag{11c}$$

$$C\equiv(4+3\,d+5\,d^2)/(1+d) \tag{11d}$$

given in [19] is then considered here. The analytical approximation L_a to the local flow component L is defined within the entire flow region $0\le d$, and moreover only involves ordinary functions of real arguments. This approximation evidently is considerably simpler than the integral representation (9).