European Journal of Mechanics B/Fluids 55 (2016) 170–181

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

# Linear disturbances in the rotating-disk flow: A comparison between results from simulations, experiments and theory

E. Appelquist [a,*], Shintaro Imayama [a], P. Henrik Alfredsson [a,*], P. Schlatter [a], R.J. Lingwood [a,b]

[a] Linné FLOW Centre, KTH Mechanics, Royal Institute of Technology, S-100 44 Stockholm, Sweden

[b] Centre for Simulation and Applied Mechanics, School of Engineering and Materials Science, Queen Mary University of London, Mile End Road, London E1 4NS, UK

ARTICLE INFO

*Article history:*
Received 27 February 2015
Received in revised form
25 August 2015
Accepted 15 September 2015
Available online 6 November 2015

*Keywords:*
Linear stability theory
Direct numerical simulations
Hot-wire anemometry
Rotating-disk boundary layer

ABSTRACT

The incompressible Navier–Stokes equations have an exact similarity solution for the flow over an infinite rotating disk giving a laminar boundary layer of constant thickness, also known as the von Kármán flow. It is well known now that there is an absolute instability of the boundary layer which is linked to transition to turbulence, but convective routes are also observed. It is these convective modes that we focus on here. A comparison of three different approaches to investigate the convective, so called Type-I, stationary crossflow instability is presented here. The three approaches consist of local linear stability analysis, direct numerical simulations (DNS) and experiments. The 'shooting method' was used to compute the local linear stability whereas linear DNS was performed using a spectral-element method for a full annulus of the disk, a quarter and 1/32 of an annulus, each with one roughness element in the computational domain. These correspond to simulating one, four and 32 roughness elements on the full disk surface and in addition a case with randomly-distributed roughnesses was simulated on the full disk. Two different experimental configurations were used for the comparison: i) a clean-disk condition, i.e. unexcited boundary-layer flow; and ii) a rough-disk condition, where 32 roughness elements were placed on the disk surface to excite the Type-I stationary vortices. Comparisons between theory, DNS and experiments with respect to the structure of the stationary vortices are made. The results show excellent agreement between local linear stability analysis and both DNS and experiments for a fixed azimuthal wavenumber (32 roughnesses). This agreement clearly shows that the three approaches capture the same underlying physics of the setup, and lead to an accurate description of the flow. It also verifies the numerical simulations and shows the robustness of experimental measurements of the flow case. The effects of the azimuthal domain size in the DNS and superposition of multiple azimuthal wavenumbers in the DNS and experiments are discussed.



## 1. Introduction

The rotating-disk flow has been of particular interest over the last century due to von Kármán's successful derivation of an exact similarity solution of the incompressible Navier–Stokes equations in cylindrical coordinates [1]. The flow belongs to a family of the so-called 'BEK boundary layers', including the Bödewadt, Ekman and von Kármán rotating boundary layers. These are characterized by a Rossby number ($Ro$), the definition of which is given in e.g. Ref. [2]. For the von Kármán boundary layer the Rossby number is $Ro = -1$. Within the flow a thin three-dimensional boundary layer is generated by the rotation of the disk, and the laminar similarity solution is described by the three velocity components: $U$, $V$ and $W$, in the radial, azimuthal and wall-normal directions, respectively, as depicted in Fig. 1. In this global picture, $U$ and $V$ increase linearly with radius ($r$) whereas $W$ is constant in $r$, indicating that the boundary-layer thickness does not change as a function of the radial position. Using the viscous length scale $L^* = (\nu^*/\Omega^*)^{1/2}$, where $\nu^*$ is the kinematic viscosity, $\Omega^*$ the angular velocity and superscript $*$ denotes a dimensional value, the Reynolds number is the nondimensional radius; $R = r = r^*/L^*$, where $r^*$ is the dimensional local radius. In Fig. 1 it is also possible to see that the radial velocity profile includes an

* Corresponding authors.
*E-mail addresses:* ellinor@mech.kth.se (E. Appelquist), phal@mech.kth.se (P.H. Alfredsson).

http://dx.doi.org/10.1016/j.euromechflu.2015.09.010

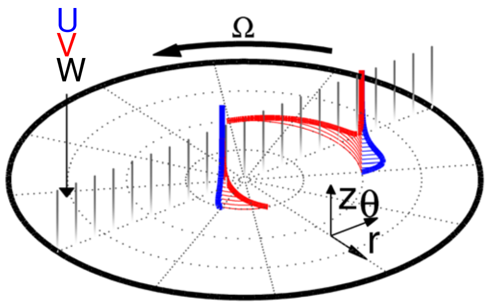


**Fig. 1.** The laminar velocity profiles of the similarity solution for the flow over a rotating disk. $U$ is the radial velocity component, $V$ is the azimuthal velocity component and the vertical greyscale lines indicate the amplitude of the wall-normal velocity component $W$ (white denotes zero velocity). The cylindrical coordinates are given by $r$, $\theta$ and $z$, and the rotation rate is defined by $\Omega$.

inflection point, which according to the Rayleigh stability criterion causes an inviscid instability. The existence of the exact similarity solution for the laminar flow led to a variety of possible approaches for analysing the flow further, creating a canonical model for other similar three-dimensional boundary layers, such as the flow over a swept wing. However, it is not only useful as a simple model of three-dimensional boundary layers but also to investigate its direct applications to rotating-flow configurations, such as turbomachinery, computer storage devices and chemical vapour deposition.

More than half a century ago experimentalists first found vortex structures in the rotating-disk boundary layer spiralling outwards with radius (e.g. Refs. [3–6]). These vortices are stationary within the rotating reference frame and correspond well to the convectively unstable stationary disturbances predicted by local linear stability analysis when using the viscous 6th-order perturbation equations (e.g. Refs. [7,8]). The vortices themselves are excited by small roughnesses on the disk surface, which are virtually unavoidable in experiments. Two different stationary modes exist that are unstable for certain parameter sets: the inviscid Type-I mode, so-called crossflow instability; and the viscous Type-II mode. The latter is attributed to the centrifugal and Coriolis forces, and has a higher critical Reynolds numbers than Type-I for stationary disturbances. Travelling Type-I and Type-II modes are also unstable over certain parameter ranges. Both stationary and travelling disturbances have been investigated by local linear stability analysis (e.g. Refs. [4,7,9–13]). Furthermore, there is a third mode, Type-III, which is a damped upstream-travelling mode that coalesces with the Type-I mode at higher Reynolds numbers. This coalescence results in the rotating-disk boundary-layer flow becoming locally absolutely unstable for some travelling disturbances above $R > 507$ [12,14]. Theoretical studies have been extended to take into account the flow development in the radial direction through global stability analysis. Pier [15] showed that the rotating-disk flow becomes nonlinearly globally unstable at the onset of local absolute instability.

This paper focuses on the Type-I crossflow stationary vortices and, even though there has been much research done both theoretically and experimentally in connection to these, only one set of direct numerical simulations (DNS) can be found in the literature. Davies and Carpenter [16] validated their linear velocity–vorticity discretization scheme by simulating Type-I and Type-II travelling waves excited by a radially-localized time-periodic wall displacement. A comparison with local theory with a stationary disturbance of azimuthal wavenumber 32 was also performed. They found that the locally-defined radial wavenumbers and growth rates from quasi-parallel linear stability theory were similar to the global linear DNS. This is also in accordance with the local–global investigation by Malik and Balakumar [17].

The purpose of this study is to compare local linear stability analysis with numerical simulations and experiments, and to establish how well they correspond to each other. In Section 2, technical descriptions of the theoretical, numerical and experimental methods are described. Linear simulations are presented to elucidate the global behaviour for disturbances triggered by regular and random distributed roughness elements, and results from two experiments will be presented, with and without deterministic surface roughness elements on the disk surface. In Section 3, the results from the comparison of the three approaches are given, which are then summarized in Section 4.

## 2. Description of the methods

### 2.1. Local linear theory

The description of the rotating-disk flow originates from the Navier–Stokes equations formulated in a cylindrical-coordinate system in the rotating frame of reference. The similarity equations for the mean flow, as formulated by von Kármán [1], were here solved numerically via a shooting method. Also, the perturbation equations that govern the linear local stability of the rotating-disk flow have been derived and solved. For the perturbation equations, the parallel-flow approximation is used to simplify the system of partial differential equations (PDE) to a sixth-order system of ordinary differential equations (ODE). This simplification means that spatial-development of the flow is neglected and only the local stability behaviour can be determined from this approach. It is possible to reduce the set of equations further by neglecting the Coriolis and streamline-curvature terms, which leads to the familiar Orr–Sommerfeld equation. However, in this context only the sixth-order system of ODE will be considered in line with the work of Lingwood [8], including rotation and curvature.

The 'shooting method' was used to investigate the local linear stability of this flow. This method includes a coordinate transformation before the normal-mode approximation is applied following the path of Lingwood [8]. For a stationary mode (in the rotating reference frame) considered here, the temporal frequency is zero ($\omega_r = 0$) and the disturbance is assumed to have the shape

$$\boldsymbol{\phi}(r, \theta, z) = \boldsymbol{\psi}(z) \exp[i(\alpha r + \beta\theta)] \tag{1}$$

for the disturbance vector $\boldsymbol{\phi} = (u, v, w, p)^T$ and the amplitude vector $\boldsymbol{\psi} = (\hat{u}, \hat{v}, \hat{w}, \hat{p})^T$, where the hat symbol denotes the spectral representation of the perturbation fields, and $\alpha$ and $\beta$ are the radial and azimuthal wavenumber, respectively. The neutral stability curves for such a disturbance are shown in Fig. 2. The curves show the boundaries in terms of $\alpha_r$, $\beta$ and the angle

$$\varepsilon = \tan^{-1}(\bar{\beta}/\alpha_r), \tag{2}$$

where $\bar{\beta} = \beta/r$, along which $\alpha_i = 0$ ($\beta$ is real by definition and $\omega_i = 0$ as a spatial analysis is performed). Within the curves $\alpha_i$ is negative and the disturbances are growing in the positive $r$-direction. Two branches are marked corresponding to the Type-I and II disturbances. For the local theory data shown in the result section of this paper, we have chosen to focus on the Type-I stationary vortices because they have higher spatial growth rates than Type-II. This difference is made clear in Fig. 3 where $-\alpha_i$ of Type-I, II and III stationary disturbances are plotted for $\beta = 32$.

The Type-III disturbance is an upstream mode, i.e. the group velocity is negative, that never becomes unstable for the stationary waves for these $R$ and thus is not seen in the neutral curves in Fig. 2. Due to the difference in group velocity, the downstream modes, Type-I and II, are unstable for $\alpha_i < 0$ in contrast to Type-III for which instability occurs for $\alpha_i > 0$. The local theory data have also previously been compared with the results from parabolic stability equations (PSE), see [17,18], where the global PSE growth-rate data were found to be slightly higher than the local theory data for Type-I and II disturbances.