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Practical mapping of the draw resonance instability in film casting of Newtonian fluids

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HIGHLIGHTS

- The draw resonance effect in film casting is quantitatively analysed.
- A Newtonian model including inertia and gravity effects is used.
- Using fluidity and inlet velocity as dimensionless control parameters reveals non-monotonic stability behaviour.
- Various stability regimes are identified, including a regime of unconditional stability.
- Correlations between the critical draw ratio and the control parameters are given.

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ABSTRACT

The influence of viscosity and inlet velocity on the draw resonance instability of film casting processes is quantitatively analysed. By linear stability analysis of a Newtonian model including inertia and gravity effects, stability curves for different control parameter values are calculated numerically. For this purpose, we propose a scaling law which separates the fluidity, i.e. the reciprocal viscosity and the inlet velocity into two independent dimensionless parameters. This new scaling evidences a minimum of stability, separating two regimes of opposite behaviour: one for which increasing the inlet flow rate has a destabilizing effect due to viscosity and one for which increasing the inlet flow rate has a stabilizing effect due to gravity and inertia; increasing the fluidity has always a stabilizing effect. By fitting the stability curves with an appropriate postulated function, we are able to construct correlations between the critical draw ratio, the fluidity and the inlet velocity. For the first time regimes of negligible inertia or negligible gravity effects are revealed as well as a regime of unconditional stability. The proposed correlations for each of these regimes can further be used as an analytical solvable criterion for determining the onset of draw resonance in film casting.

1. Introduction

The process of film casting is of great importance in polymer and glass industry. In general, material, which is mostly in a molten state, is extruded through a slit die and drawn at higher speed, such that its thickness becomes thinner. Exceeding a critical take-up velocity, or equivalently exceeding the so-called critical draw ratio, oscillation in both flow velocity and width can be observed, which leads to minor quality of the end-product and eventually to the breakdown of the process. Miller [1] observed this phenomenon for the first time in fibre spinning and named it as draw resonance.

Since then, a lot of work has been done on the description of film casting processes, most of the studies focusing on the prediction of draw resonance, some including three-dimensional effects [2]. A comprehensive review is given in Chapter 10 of the book edited by Hatzikiriakos and Migler [3]. Yeow [4] has been the first who applied linear stability analysis on the process of isothermal film casting of a Newtonian fluid, neglecting all secondary forces like inertia, gravity, surface tension and air drag.

Shah and Pearson [5] investigated the effect of secondary forces on the stability of fibre spinning for the first time.[1] Newer results regarding the effect of inertia and gravity in film casting have been published by Cao et al. [6]. Using linear stability analysis, they investigated the dependence of the critical draw ratio and the oscillation frequency on Reynolds number *Re* and Froude number *Fr*. Moreover, a nonlinear analysis has been carried out in order to examine the influence of inertia and gravity on the oscillation amplitudes, film rupture and the time needed to reach sustained

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[1] Besides a numerical factor, two-dimensional film casting and fibre spinning are mathematically equivalent, as will be also mentioned in Section 2.

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oscillation. One of their main conclusions is that both gravity and inertia effects improve stability, with inertia being the dominant force regarding stability. However, up to now a quantitative correlation between experimental processing parameters and the onset of instability is still missing. Cao et al. [6] considered a parameter range of Re $\in [0, 0.25]$ and Re/Fr $\in [0, 25]$. This covers a wide practical range, but as presented here, it still misses experimental important parameter sets, including a regime where the instability vanishes completely.

Besides full numerical simulations some approximating methods have been developed, which greatly facilitates the computation of the critical draw ratio with respect to the system parameters. One of such alternative approaches is Hyun's stability criterion [7], later adapted by Kim et al. [8], which correlates the cycle time of draw resonance with the travelling time of a mass element through the system. Hagen published several mathematical analysis on draw resonance, including some works on the asymptotic behaviour of the eigenvalue spectrum of the linearised solutions for both isothermal [9] and non-isothermal [10] models. Moreover, Van der Hout [11] derived another stability criterion for fibre spinning of Newtonian and power law fluids. However, to our knowledge no correlations between the critical draw ratio and process and material parameters including the effects of inertia and gravity have been revealed yet.

In our work, the inlet velocity and the fluidity, i.e. the reciprocal viscosity, are used as control parameters as it is the case in practical settings. We start with a short presentation of the common model equations and introduce a scaling rule which facilitates the examination of the influence of the control parameters on the onset of the instability. This is followed by a description of the stability analysis method, including the postulation of an analytical solvable expression for the stability curves, which can be fitted to the numerical data and used as an estimator for the critical draw ratio. As a result, we can identify regimes where gravity, or inertia respectively, can be neglected, as well as a regime of unconditional stability. These are all summarised in a stability map covering the practical range of the control parameters. The paper closes with conclusions and outlook.

2. Model equations and scaling

We follow the usual procedure of modelling the film casting process for stability analysis, as presented for example by Cao et al. [6]. Fig. 1 shows the three-dimensional sketch of a film which is drawn along the x-axis over a total distance L. In this work a one-dimensional model is used, using the assumption of infinite width. Under the assumption that the film thickness $h(x, t)$ is small compared to L, i.e. $h/L \ll 1$, the flow velocity in x direction, denoted by u, does not change across the thickness at leading order, i.e. $u = u(x, t)$, t being the time. The continuity equation can then be written,

$$\partial_t h + \partial_x(hu) = 0. \tag{1}$$

Following Yeow [4], we use a Newtonian constitutive equation. The momentum balance equation then has the following form, neglecting surface tension and air drag[2]:

$$\rho h\,(\partial_t u + u\partial_x u) = \partial_x\,(4h\eta\partial_x u) + g\rho h, \tag{2}$$

where the left-hand side accounts for inertia and the two terms on the right-hand side account, respectively, for viscous stresses and gravity; ρ and η are, respectively, the density and the dynamical

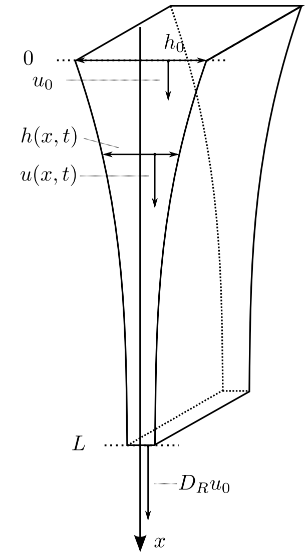

Fig. 1. Sketch of film casting process.

viscosity of the material, both assumed to be constant, and g is the acceleration of gravity. The factor 4 in Eq. (2) is the so-called 'Trouton ratio'.

Within the process of film casting, material is extruded with a certain velocity u_0 through a slit die of thickness h_0 and taken up by a chill roll rotating with predefined speed. This sets the boundary conditions, which are

$$u(0, t) = u_0, \qquad u(L, t) = D_R u_0, \qquad h(0, t) = h_0, \tag{3}$$

where D_R is the so-called draw ratio, strictly larger than unity.

In this work, we are considering that the length L is fixed and that the adjustable control parameters, in addition to the draw ratio, are the inlet velocity and the viscosity. The former can be changed by different extrusion dies or different flow rates, the latter by modifying the temperature or the material itself. In order to get dimensionless variables, we are therefore using the following transformation rules:

$$x \to Lx, \qquad u \to \sqrt{gL}\,u, \qquad h \to h_0 h, \qquad t \to \sqrt{\frac{L}{g}}\,t. \tag{4}$$

Applying the transformations (4) to (1) and (2) leads to the following system:

$$\partial_t h + \partial_x(hu) = 0, \tag{5a}$$

$$Fh(\partial_t u + u\partial_x u - 1) - \partial_x(h\partial_x u) = 0, \tag{5b}$$

where $F = \frac{\sqrt{gL^3}\rho}{4\eta}$ is the fluidity parameter, i.e. the dimensionless reciprocal viscosity. The boundary conditions in (3) become

$$u(0, t) = Q, \qquad u(L, t) = D_R Q, \qquad h(0, t) = 1, \tag{6}$$

where $Q = \frac{u_0}{\sqrt{gL}}$ is the dimensionless inlet velocity. The pure viscous model reported in [4] is recovered by setting $F = 0$, for any $Q \neq 0$; for convenience, we have set $Q = 1$. It is worth mentioning that all the equations mentioned here can merely be transferred to the fibre spinning model first proposed by Matovich and Pearson [12] by substituting the thickness $h(x, t)$ with the cross-sectional area of the fibre and additionally changing the Trouton ratio 4, which appears in F, to 3. Nevertheless, surface tension effects induced by transverse curvature variations along the fibre have to be considered in addition to gravity and inertia.

Notice that the velocity scale $\sqrt{gL}$ is the characteristic velocity of a fluid particle falling along the length L under gravity. For a

[2] As indicated by a referee, these assumptions may be inappropriate for modelling of fibre spinning, limiting the comparability of these results with the fibre spinning process.