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Numerical and experimental modeling of melt flow in a directional solidification configuration under the combined influence of electrical current and magnetic field

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ABSTRACT

One of the key issues in the directional solidification (DS) process of multi-crystalline silicon is the control of the melt flow in order to achieve a higher quality of the crystallized material. The combination of a static magnetic field B and an electrical current I, giving rise to an electromagnetic force has a significant melt stirring effect, even for small values of I and B. In order to understand the basic features of the melt flow in a DS-like configuration under electromagnetic stirring, an isothermal model experiment in a rectangular crucible filled with a room temperature GaInSn melt and a corresponding STHAMAS3D time-dependent numerical model, were developed. Experimental velocity profiles measured by UDV confirmed the flow structure obtained in the numerical simulations. A parametrical study for a range of I and B values was performed, in the case of a symmetrical electrode positioning along the diagonal of the free melt surface. The resulting flow structure was analyzed and described in terms of a vortex or a poloidal recirculation domination and a transition between the two. A characteristic parameter was defined to quantify the different flow structures. Through the use of scaling analysis, two dimensionless numbers corresponding to the two Lorentz force components were identified and a good correlation between their values, flow structure and maximal velocity was observed. This correlation makes possible the prediction of the flow structure for any set of the system parameters I and B and a characteristic crucible length. The same conclusions would hold for a silicon melt if the dimensionless numbers are conserved by choosing different I and B in respect with the different material constants.

1. Introduction

One of the key issues in the directional solidification process of multicrystalline silicon is to control the melt flow in order to obtain a more uniform distribution of impurities in the melt and a proper interface shape which, in turn, will lead to a higher quality of the crystallized material. The costs of DS of silicon can be decreased by increasing the crucible size, which can lead to increasing yield and throughput of a production line. Possible problems can arise, like turbulent flows and the precipitation of impurities coming from the feedstock, crucible or furnace environment in poorly mixed areas. Magnetic fields are a powerful tool to control the convection in electrically conducting melts and, therefore, play a key role in the optimization of semiconductor bulk crystal growth processes [1,2]. Even though, at the moment, magnetic fields are not used on a large scale in the industrial directional solidification process of multicrystalline silicon, there is a huge ongoing research effort to show the potential of the magnetic fields for the control of melt convection in this process. There are two main ideas developed in literature: one based on traveling magnetic fields (TMF) [3–8] and the other based on a combination of a static magnetic field and an electrical current (EMF) [9–11]. It was shown that in the case of EMF, even small values of the electrical current (2–10 A) combined with a low magnetic field (10 mT) can have a beneficial effect on the melt convection, interface shape and chemical species mixing inside the silicon melt, preventing impurity precipitation or turbulent flow.

Time-dependent magnetic fields (e.g. TMF), have limited mixing efficiency in large melts, caused by magnetic field damping due to the skin depth effect. The advantage of the EMF is the high efficiency stirring in the whole melt volume [10].

The main contribution of the EMF stirring method is to provide some additional growth parameters (like the intensity of the

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Table 1
Material properties of the GaInSn eutectic melt [12–14].

Physical property	Value	Unit
Density, ρ	6360	kg/m^3
Dynamic viscosity, μ	$2.16 \cdot 10^{-3}$	$N\,s/m^2$
Electrical conductivity, σ	$3.2 \cdot 10^{6}$	S/m

electrical current and the position of the electrodes) easy to adjust in order to control the melt flow and interface shape. A deep understanding of the influence of EMF parameters on the melt convection is crucial for the optimization of the ingot quality. In order to understand the basic features of the melt flow in a DS-like configuration under EMF, we propose in the present work a model experiment and a corresponding 3D time-dependent numerical model for the melt flow.

The model experiment consists of a square-shaped crucible placed in a vertical magnetic field. The crucible contains a GaInSn room temperature liquid alloy, which is similar to molten silicon. Two electrodes are in contact with the alloy melt surface, through which a DC electrical current passes. STHAMAS3D numerical simulations were performed for the same conditions. Velocity profiles perpendicular to the crucible surface were measured by Ultrasound Doppler Velocimetry (UDV, or UVP, Ultrasound Velocimetry Profile, as sometimes mentioned in the literature), in order to validate the simulation results by comparing them with the experimental ones. To have a better understanding on the influence of electromagnetic stirring on impurity distribution and growth interface in DS of multicrystalline silicon, one must first understand, quantify and, ideally, predict the different flow regimes that may arise from this type of melt stirring for a given system configuration, when changing the electromagnetic parameters. Therefore, we have performed a parametrical study on the melt flow by making a large number of numerical simulations for the EMF stirring of an isothermal GaInSn melt with an aspect ratio close to that of a G1 silicon ingot and the case of a symmetrical electrode positioning. The present results can be of interest also for other fields in materials science and metallurgy where the melt stirring is necessary, like stirring floating particles into liquid metal or flow control in DC electric arc furnaces.

2. Numerical model

The geometry used in the numerical modeling consists of a rectangular crucible, $7 \times 7 \times 5$ cm^3 in size. The crucible is filled with an eutectic GaInSn alloy. This alloy, because of its low melting point temperature (~11 ° C) is liquid at room temperature and is usually used as a model fluid in experiments. The material properties used in the numerical modeling are listed in Table 1:

An electrical current is injected in the melt through two electrodes placed along the diagonal symmetrically from the center point, at 1/3 and 2/3 of the diagonal length from the lower left corner, as can be seen from Fig. 1.

This set-up is placed in a vertical magnetic field. The combination of the electrical current and the vertical magnetic field generates a Lorenz force which gives a rotational flow component to the fluid convection.

The mathematical model is described by the Navier–Stokes equations for an incompressible fluid:

$$\nabla \cdot u = 0 \tag{1}$$

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho (\boldsymbol{u} \cdot \nabla) \boldsymbol{u} = -\nabla p + \mu \Delta \boldsymbol{u} + \boldsymbol{s_u} \tag{2}$$

where ρ is the fluid density, $\boldsymbol{u}$ is the flow velocity, p is the pressure and $\boldsymbol{s_u}$ represents the source term for the momentum equation. In order to simulate the influence of the magnetic field on the fluid flow, the Lorentz force density is included in the source term. In this case:

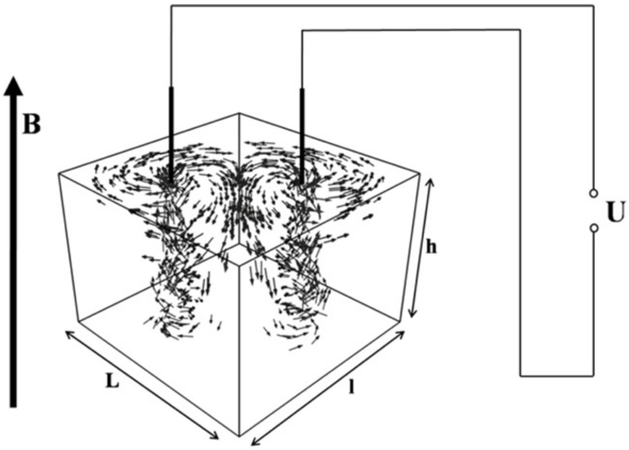

Fig. 1. Schematic representation of the numerical model.

$$\boldsymbol{s_u} = \boldsymbol{F_L} = \boldsymbol{j} \times \boldsymbol{B}. \tag{3}$$

The electrical current density, $\boldsymbol{j}$, can be determined from Ohm's law:

$$\boldsymbol{j} = \sigma (\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) \tag{4}$$

where σ is the electrical conductivity.

The Lorentz force density is obtained by combining Eqs. (3) and (4), where the electrical field $\boldsymbol{E}$ is obtained from a scalar potential Φ:

$$\boldsymbol{E} = -\nabla \Phi. \tag{5}$$

By introducing this approximation for the electrical field, an additional equation for the scalar electrical potential needs to be solved. This equation derives from the continuity equation for the current density:

$$\nabla \cdot \boldsymbol{j} = 0 \tag{6}$$

which leads to the governing equation for the scalar potential Φ:

$$\Delta \Phi = \nabla \cdot (\boldsymbol{u} \times \boldsymbol{B}) . \tag{7}$$

Therefore, the Lorentz force density can be written as:

$$\begin{aligned} \boldsymbol{F_L} &= \sigma (-\nabla \Phi + \boldsymbol{u} \times \boldsymbol{B}) \times \boldsymbol{B} \\ &= \sigma (-\nabla \Phi) \times \boldsymbol{B} + \sigma (\boldsymbol{u} \times \boldsymbol{B}) \times \boldsymbol{B}. \end{aligned} \tag{8}$$

The crucible walls and the melt free surface are considered to be electrically isolated ($j_n = 0$) except for the elements at the free surface where the two electrodes are in contact with the fluid:

$$\nabla|_n \Phi = -\frac{\boldsymbol{j_n}}{\sigma} \tag{9}$$

where $\boldsymbol{j_n}$ is the density of the electrical current that passes through the two electrodes.

From Eq. (8), one can see that the Lorentz force has two major components. The first component is $\sigma(-\nabla \Phi) \times \boldsymbol{B}$, which is proportional to $I \cdot B$, since it represents the product of the potential gradient, which at an electrode is the imposed current density, with the applied magnetic field B. Around the electrodes, where the flow originates and the Lorentz force is maximum, the current density decreases exponentially from an initial value always proportional to the imposed electric current intensity I. This component is the accelerating component of the Lorentz force, responsible for driving the flow. It always appears in the horizontal plane due to the cross product of the vertical magnetic field induction $\boldsymbol{B}$ with the radial component of the electric