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The dynamics of cylinder in a confined swirling flow with constant vorticity

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ABSTRACT

In this work, we present a rigid body dynamics model that accounts for phenomena earlier studied both within hydrodynamic stability theory and the area of fluid-induced vibrations. The model captures the transverse dynamics of a rigid cylinder in a confined swirling flow. We show that a linear inviscid stability analysis of the whole system with respect to two-dimensional disturbances could be decomposed into a solution governing rotational disturbances with homogeneous boundary conditions and a solution governing irrotational disturbances with inhomogeneous boundary conditions. This implies that the continuous stable spectrum of rotational disturbances is unchanged by the supposition of a free boundary. Moreover, the time-dependence of irrotational disturbances is governed by the disturbance of the rigid cylinder. Consequently, a rigid body dynamics model suffices to determine the time evolution of irrotational disturbances. The model is based on the definition of a merged homogeneous state in which the solid mass of the rigid cylinder equals the displaced fluid mass and the flow is in solid body rotation. A departure from this merged homogeneous state yields an imbalance of the fictitious Coriolis and centrifugal force of the rigid cylinder and the counterbalancing motion-induced fluid forces. This imbalance makes the fluid flow support propagation of waves and may render a concentric position of the body unstable. A non-uniform distribution of the angular velocity delays the onset of instability so that the rigid cylinder can maintain a concentric position even though it is denser than the fluid.

1. Introduction

Let us consider a swirling flow with uniform vorticity between concentric cylinders. The outer cylinder is fixed in space and the inner cylinder is free to move in the transverse plane. The inner cylinder must coincide with the material surface of the inner boundary of the fluid flow. The stability of the system is therefore both a matter of the stability of the fluid flow and the stability of the inner cylinder. It is well known that a normal-mode analysis of two-dimensional rotational disturbances of a flow with uniform vorticity between concentric fixed cylinders, i.e. homogeneous boundary conditions, does not yield any discrete mode but a continuous spectrum of stable singular modes (see for example Drazin and Reid [1]). However, the supposition of a free surface yields discrete azimuthal modes and the existence of irrotational disturbances in the fluid flow. Phillips [2] observed such modes as waves on the inner free surface in an experimental set-up of a partially filled horizontal rotating cylinder. He theoretically determined the modes induced by a steady disturbance of the free surface caused by gravity. After showing that the fictitious centrifugal force plays the same role as the gravity force in gravity waves, Phillips called these waves on the inner free surface, centrifugal waves. The special case of two-dimensional waves with one node in the plane perpendicular to the axis of the cylinder corresponds to a rigid body motion of the free surface. These transverse waves travel forward and backward with respect to the rotating flow. This case was also treated by Miles and Troesch [3] in a study of free surface oscillations of a partially fluid-filled rotating vertical cylinder in which gravity was neglected. The surface oscillations may render a partial fluid-filled rotating cylinder unstable (see for example [4, p. 539]).

This raises the question, could centrifugal waves also occur in a system of concentric cylinders where the outer cylinder is fixed and the inner cylinder can move only in the transverse plane? What happens if the inner free surface in the above examples is replaced with a rigid cylinder? Would for example a rotor immersed in a confined swirling flow experience transverse oscillations corresponding to the transverse waves of one node?

In connection to different engineering problems, the dynamics of a rotating cylinder surrounded by an annular swirling flow

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has been extensively studied. In simplified analytical models of such problems, the aim is often to determine the motion-induced fluid forces on the cylinder. However, most analytical studies that include inertial effects have been limited to the narrow-gap approximation based on bulk-flow models in which the pressure build-up of the flow induced by the radial acceleration of the basic flow is neglected. Fritz [5] studied the stability of a rotor surrounded by turbulent flow and is among the earliest work with such an approach.

A static eccentric position of a cylinder in a confined swirling flow yields a static force that pulls the cylinder towards the wall which was deduced by Milne-Thomson [6, p. 181] for an irrotational swirling flow with constant circulation. Such a suction force is referred to by Brennen as the Bernoulli effect [7]. Antunes [8] discusses this effect in a study on the dynamics of a rotor immersed in eccentric annular flow. The theoretical results are based on a bulk-flow model in which the circumferential velocity is assumed to be constant. The results show that as the gap width increases and the inertial effects dominate, the static force of an eccentric position changes from pushing the rotor at an angle with respect to the eccentric position to pushing it in the direction of the eccentricity. Still, these two examples are based on different assumptions. The model of Milne-Thomson assumes that the flow field has a varying circumferential velocity distribution and an arbitrary gap width whereas a bulk-flow model is based on the narrow-gap approximation and the circumferential velocity is assumed to be constant along the gap width.

The B/M model introduced by Muszynska and intended to explain self-excited vibrations of rotor includes a rotating inertia fluid force [9]. The fluid model is here based on earlier work based on a bulk-flow model. The rotating inertia force gives rise to a negative elastic force, i.e. a suction force. However, Jansson [10] showed that as the gap width increases additional inertial effects of the fluid flow arise that counterbalances the suction force. Another work that also includes these additional effects was published by Brennen in 1976 [11]. He derived general solutions for the flow of an annulus surrounding a whirling cylinder and deduced the motion-induced fluid forces for a number of special cases. However, the whirling motion of the cylinder is assumed to be constant and forced on the fluid flow. The question is left open what the effect is of these fluid-induced forces on the dynamics of the cylinder.

Jansson et al. [12] carried out a theoretical analysis on the dynamics of a cylinder supported on hinges surrounded by a confined swirling flow with no vorticity. Vladimirov and Ilin's work [13] deals with the stability of systems that consist of both a solid and a fluid. For this purpose, they developed an extended version of Arnold's method. In one of their examples, they show that a system of a cylinder surrounded by annular flow in solid body rotation is unstable if the cylinder is denser than the fluid. Kop'ev and Chernyshev [14] studied the instability of a cylinder in an unbounded circulation flow.[1] They obtained exact analytical results for two different choices of vorticity distributions. Regions of instability were derived for certain values of the density ratio and natural frequency of the oscillating cylinder. The first case has a monotonically decreasing vorticity and the second one has an inner layer in solid body rotation, i.e. constant vorticity and an outer layer with constant circulation, i.e. no vorticity. They stated that the cylinder could only be unstable in the presence of vorticity. Still, it remains unclear how a departure from a state of solid body rotation changes the onset of instability. This issue is discussed in this work. Furthermore, we discuss the effect on the gap width on the stability. In the presence of an outer boundary, can a concentric position of the cylinder be unstable even in the case of constant circulation, i.e. no vorticity?

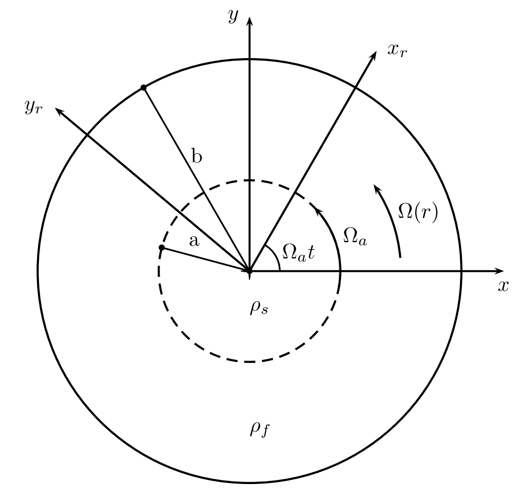

Fig. 1. The basic flow has an angular velocity distribution, $\Omega(r)$, with constant vorticity Z. The rotating frame of reference rotates with the angular velocity of the material equal to Ω_a.

In this work we solve the stability problem with respect to two-dimensional disturbances of a flow with uniform vorticity between concentric cylinders. The outer cylinder is fixed and the inner cylinder is free to move in the transverse plane. The main contribution of this work is the presentation of a rigid body dynamics model. We show that rotational disturbances are independent of the supposition of a free surface as the inner boundary and that irrotational disturbances are governed by the dynamics of the inner cylinder. The stability problem hence reduces to a rigid body dynamics problem.

The results are interpreted in terms of the centrifugal and Coriolis force of the solid mass and the displaced fluid mass. If the flow rotates a as solid body and the fluid and the solid are of equal density, the motion-induced fluid forces cancel out the fictitious forces of the solid mass. This state is referred to as a merged homogeneous state. A departure from this state yields either stable transverse oscillations of the body or instability. The stability of the rigid body is given as a function of the following three parameters; the radius ratio, the vorticity ratio and the density ratio.

2. Problem formulation

Imagine an inviscid fluid confined by concentric cylinders of radii a and b, (see Fig. 1). The outer cylinder is fixed in space whereas the inner cylinder is free to move in the transverse plane. The inner cylinder is a rigid body with a solid mass per unit length equal to $m_s = \rho_s \pi a^2$. The displaced fluid mass is equal to $m_D = \rho_f \pi a^2$. The basic fluid flow has a radial distribution of the angular velocity, $\Omega(r)$, that satisfies a uniform vorticity distribution equal to Z. We here consider the linear inviscid stability of this system with respect to two-dimensional disturbances. The stream function of the perturbation velocity is defined as

$$\tilde{\psi}(r, \theta, t) = \Re \left\{ \phi_n(r, t) e^{in\theta} \right\}. \quad (1)$$

The material surface that defines the inner boundary of the fluid rotates with the angular velocity, Ω_a, i.e. $\Omega(a) = \Omega_a$. The impermeability boundary condition states that the position of the rigid body must coincide with this material surface. The rigid body hence refers to the material surface as well.

The position of the body is a function of time, t, and is stated in complex coordinates. In the inertial coordinate system, the disturbance position of the body is equal to $z_s(t) = x_s(t) + iy_s(t)$.

[1] The authors are thankful to the reviewers for mentioning this reference.