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Flow rate in a channel with small-amplitude pulsating walls

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ABSTRACT

The influence of wall transverse pulsation on the viscous incompressible fluid flow in a channel under the action of pressure and gravity is studied under approximation of the small amplitudes. It is found that for the large values of the Womersley numbers the pulsations decrease the flow rate, rather than increase it, as it was found in the previous studies for the small Womersley numbers.

1. Introduction

A two-dimensional flow in the field of the gravitational force in a channel with pulsating walls in the presence of pressure drop is studied. This problem was previously analyzed as a particular case of peristaltic flow by S.L. Weinberg [1] and M.S. Longuet-Higgins [2], who illustrated that the pulsations result in increase of the flow rate, which occurs due to the pressure drop. Later, N.I. Arinchin [3] used this effect to explain the significant increase of blood flow in the working muscles. Building the research by G.V. Anrep et al. [4], he proposed that the muscles with high frequency vibrations squeeze and relax dependent vessels and as the result, the increase of flow rate is observed. G.A. Lyukhov and I.V. Shugan [5,6] suggested that the effect found by S.L. Weinberg [1] and M.S. Longuet-Higgins [2] could be used to improve the pumps performance and they have calculated the energy consumption required to increase the flow rate by means of pulsations. K. Lee et al. [7] arrived at a differential equation for the flow rate on the bases of the assumptions, some of which (for example, an assumption about local Poiseuille profile of longitudinal velocity) being not evident. K. Lee et al. [7] came to the conclusion that the flow rate increases with the increase in the amplitude of the walls pulsation and decreases with the decrease of the Womersley number (product of the distance between the walls by square root from frequency ratio to kinematic liquid viscosity). Their theory states that for the small walls pulsation amplitudes the pulsations result in the flow rate increase and the high walls pulsation amplitudes could lead to both the flow rate increase and its decrease depending on the Womersley number value.

We study the solution to the problem about the flow in a channel with pulsating walls applying the solution class of Navier–Stokes equations with a linear dependence from coordinate being longitudinal with respect to the channel axis. Earlier, T.W. Secomb [8] considered this type of solution for this problem focusing on the analysis of the case with the absent of pressure differences at the channel ends and gravity. The research of T.W. Secomb illustrates the existence of nonzero time-averaged velocity field of flow; this flow, however, does not contribute into the time-averaged flow rate. Later P. Hall and D.T. Papageorgiou [9] considered the stability of this solution. Their numerical calculations showed that the increase of the Womersley number for high amplitudes of walls pulsations led to the chaos according to the Feigenbaum scenario via the cascade of period doubling.

The problem about the channel walls pulsation influence on the solute concentration distribution and the problem about the squeezing flow are close to the ones considered in this article. S. Tsangaris [10] was among the first studies on the first problem. Later S.L. Waters [11] looked at this problem and found out that the solute flow increased as the diffusion coefficient was decreasing, the Womersley number was increasing and the walls permeability coefficient was increasing. Going deeper into the problem S.L. Waters [12] presented a mathematical model of oxygen supply to the heart after laser myocardial revascularization.

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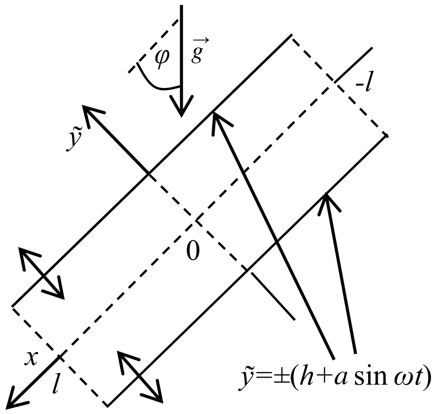

Fig. 1. Geometry of the problem.

The problem of the squeezing flow is characterized by the approaching walls squeezing liquid from the channel. For some laws of walls movements there are analytical solutions to this problem [13–15]. K.J. Zwick et al. [16] experimentally studied the squeezing flow. They analyzed the flow of Newtonian and viscoplastic liquids between two round plates, which were under the acting force comprising oscillating and constant components. In case of viscoplastic liquid they found that the plates vibration resulted in significant increase of flow rate, while no such effect was found in case of Newtonian liquid.

The purpose of the present paper is to examine the influence of channel walls pulsations on the fluid transport for the large Womersley numbers. This problem is solved under the approximation of small-amplitude wall pulsation. Section 1 deals with the review on the literature. The second one describes general problem statement. Section 3 of the paper states and solves the problem in zero and first order by wall pulsation amplitude. Section 4 states the problem and its time-averaged solution in the second order by the amplitude. In Sections 5 and 6 we use the found solutions to analyze the influence of channel walls pulsations on the flow rate and time-averaged velocity field of flow.

2. Problem statement

Let us consider a two-dimensional flow of viscous incompressible fluid flow in a channel, which walls harmonically pulsate with the frequency ω and the amplitude a at the presence of the gravity and pressure difference 2Δ at the ends of the channel with the length $2l$ (Fig. 1).

The fluid flow is described by a system of Navier–Stokes equations for the viscous incompressible Newtonian fluid and by a continuity equation:

$$\rho\left(\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+v\frac{\partial U}{\partial \tilde{y}}\right)=-\frac{\partial \tilde{p}}{\partial x}+\eta\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial \tilde{y}^2}\right)+\rho g\cos\varphi, \tag{1}$$

$$\rho\left(\frac{\partial v}{\partial t}+U\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial \tilde{y}}\right)=-\frac{\partial \tilde{p}}{\partial \tilde{y}}+\eta\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial \tilde{y}^2}\right)-\rho g\sin\varphi, \tag{2}$$

$$\frac{\partial U}{\partial x}+\frac{\partial v}{\partial \tilde{y}}=0, \tag{3}$$

where x and $\tilde{y}$ are longitudinal and transverse (with the relation to the axis of channel) coordinates, t is time; U and v are longitudinal and transverse components of velocity, p is pressure; g is the acceleration of gravity, ρ—is fluid density, η is dynamic viscosity, φ—angle between the channel axis and the direction of gravity force.

The position of the upper and lower channel wall in a given time moment is determined by the equation

$$\tilde{y}=\pm(h+a\sin\omega t). \tag{4}$$

The position of the channel ends are specified by the equation

$$x=\pm l. \tag{5}$$

The walls points move only along the OY axis. No-slip boundary conditions are identified on the lower and the upper walls:

$$\tilde{y}=\pm(h+a\sin\omega t):\ U=0,\qquad v=\pm a\omega\cos\omega t. \tag{6}$$

Pressure-drop boundary condition is specified at the channel ends:

$$\tilde{p}|_{x=l}-\tilde{p}|_{x=-l}=2\Delta. \tag{7}$$

Choose the following units of measurement: length—h, time—$1/\omega$, pressure—$\eta\omega$, velocity—$h\omega$. The problem in a dimensionless form is characterized by four parameters: the Womersley number W, the Richardson number Ri, dimensionless amplitude of pulsations A and dimensionless channel length L:

$$W=h\sqrt{\frac{\rho\omega}{2\eta}},\qquad \mathrm{Ri}=\frac{g\cos\varphi-\Delta/(l\rho)}{\omega^2 h},$$

$$A=\frac{a}{h},\qquad L=\frac{l}{h}.$$

After the pressure transformation

$$\tilde{p}\to p-\rho y g\sin\varphi+x\Delta/l, \tag{8}$$

the transformation of transverse coordinate

$$\tilde{y}\to y\left(1+\frac{a\sin\omega t}{h}\right) \tag{9}$$

and the nondimensionalization the system of Navier–Stokes equations (1)–(2) and continuity equation (3) has the following form

$$(1+A\sin t)^2\left[\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\frac{v-yA\cos t}{1+A\sin t}\frac{\partial U}{\partial y}\right]=\frac{1}{2W^2}\left[\left(-\frac{\partial p}{\partial x}+\frac{\partial^2 U}{\partial x^2}\right)(1+A\sin t)^2+\frac{\partial^2 U}{\partial y^2}\right]+(1+A\sin t)^2\mathrm{Ri}, \tag{10}$$

$$\frac{\partial v}{\partial t}+U\frac{\partial v}{\partial x}+\frac{v-yA\cos t}{1+A\sin t}\frac{\partial v}{\partial y}=\frac{1}{2W^2}\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{(1+A\sin t)^2}\frac{\partial^2 v}{\partial y^2}-\frac{\partial p}{\partial y}\right), \tag{11}$$

$$\frac{\partial U}{\partial x}+\frac{1}{1+A\sin t}\frac{\partial v}{\partial y}=0 \tag{12}$$

and the boundary conditions (6)–(7) are the following ones

$$y=\pm 1:\ U=0,\qquad v=\pm A\cos t, \tag{13}$$

$$p|_{x=L}=p|_{x=-L}. \tag{14}$$

The purpose of the coordinate transformation (9) is to set the boundary conditions at the fixed boundary.

Pressure transformation (8) leads to the exclusion of summand with Δ from the boundary condition (14). At the same time the Richardson number Ri contains a member $\Delta/(L\rho)$ besides the gravity acceleration g. This transformation results in the exclusion of summand $\rho g\sin\varphi$ from the equation for transverse velocity (11).