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Unsteady boundary layer flow over a permeable curved stretching/shrinking surface

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HIGHLIGHTS

- Unsteady boundary layer flow is numerically studied.
- The flow involves a curved surface.
- Dual solutions are found and discussed along with a stability analysis.
- Different flow behavior is depicted.

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ABSTRACT

The problem of unsteady viscous flow over a curved stretching/shrinking surface with mass suction is studied. A similarity transformation is used to reduce the system of partial differential equations to an ordinary differential equation. This equation is then solved numerically using the function bvp4c from Matlab for different values of the curvature, mass suction, unsteadiness and stretching/shrinking parameters. The physical quantities of interest, such as reduced skin friction, velocity and shear stress are obtained and discussed as functions of these parameters. Results show that for both cases of stretching and shrinking surfaces, multiple (dual, upper and lower branch) solutions exist for a certain range of curvature, mass suction, unsteadiness and stretching/shrinking parameters. This is an opposite situation than that of the plane stretching sheet. In order to establish which of these solutions are stable and which are not, a stability analysis has been performed. It is evident from the results that the pressure inside the boundary layer cannot be neglected for a curved stretching sheet, as distinct from a flat stretching sheet.

1. Introduction

The flow induced by a moving boundary is important in the study of extrusion processes and is a subject of considerable interest in the contemporary literature. For example, materials which are manufactured by extrusion processes and heat-treated materials traveling between a feed roll and a wind-up roll or on conveyor-belts possess the characteristics of stretching/shrinking surfaces (Fischer [1]). Polymer sheets and filaments, wire deawing, metal spinning, etc., are also manufactured by continuous extrusion from a die to a windup roller which is located a finite distance away (Sparrow and Abraham [2]). The mechanical properties of the final product depend strictly on the stretching and cooling rates. Most of these solutions are based on the boundary layer assumption and therefore do not constitute exact solutions of the Navier–Stokes equations (Wang [3,4]). However, a closed-form exact solution for a two-dimensional laminar flow of an incompressible viscous fluid over a linearly stretching sheet has been given by Crane [5]. Gupta and Gupta [6] examined the stretching flow with heat and mass transfer in the presence of suction or injection. Banks [7] has reported similarity solutions of the boundary-layer equations for an impermeable stretching surface, while Magyari and Keller [8] for a permeable stretching surface. Further, Wang [9] has analyzed the viscous flow due to a stretching sheet with surface slip and suction. In another paper, Wang [10] extended the flow analysis to a three-dimensional axisymmetric stretching surface.

On the other hand, the study on boundary layer flow due to a shrinking sheet has also attracted much attention. It seems that Liao [11,12] pioneered the study of flow on a shrinking sheet. He obtained multiple solutions for both impermeable and permeable shrinking sheets. Also, Miklavčič and Wang [13] studied the flow on a shrinking sheet, which is an exact solution of the Navier–Stokes equations. They found that the vorticity over the

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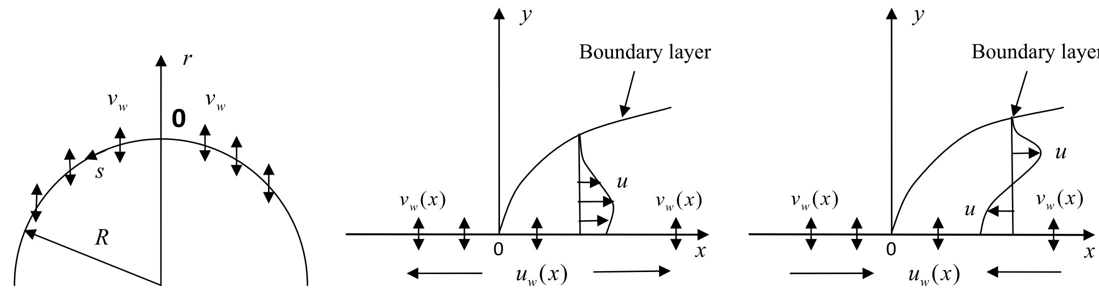

Fig. 1. Physical model and coordinate system: (a) curvature sheet; (b) stretching surface; (c) shrinking surface.

shrinking sheet is not confined within a boundary layer. To confine the velocity of the shrinking sheet in the boundary layer, Miklavčič and Wang [13] imposed an adequate suction on the boundary, while Wang [14] considered a stagnation flow. This new type of shrinking sheet flow is essentially a backward flow, as discussed by Goldstein [15] and it shows quite distinct physical phenomena. Ever since the paper by Crane [5] has been published, numerous studies have been investigating different aspects of this problem. Fang et al. [16] solved the viscous flow over a shrinking sheet analytically using a second order slip flow model. They found that the solution has two branches, or dual solutions, in a certain range of the parameters. Bhattacharyya et al. [17] observed that the velocity and thermal boundary layer thicknesses for the second solutions are always larger than those of the first solutions.

Another very important class of shrinking surfaces is the unsteady viscous flow over a continuously shrinking surface with mass suction and it has been studied by Fang et al. [18]. The solution is an exact solution of the unsteady Navier–Stokes equations. Similarity equations are obtained through the application of similarity transformation techniques and these equations were solved for different values of the mass suction parameters and the unsteadiness parameters. Results show that multiple solutions exist for a certain range of mass suction and unsteadiness parameters. Quite different flow behavior is observed for an unsteady shrinking sheet that for an unsteady stretching sheet.

In a series of papers by Sajid et al. [19,20] and Abbas et al. [21], the Crane's problem [5] was extended to a curved stretching sheet. The authors have analyzed the effects of curvature and found that its presence inside the boundary layer is no more negligible as in the case of a flat stretching sheet. Motivated from this fact, our aim here is to extend the analysis of Fang et al. [18] on the viscous flow over an unsteady plane shrinking sheet with mass transfer to a curved stretching/shrinking surface. By selecting appropriate similarity variables, the partial differential equations are transformed into ordinary (similarity) differential equations, which are then solved numerically using the function bvp4c from Matlab for different values of the governing parameters. It is found that the solutions of the ordinary (similarity) differential equations have two branches, upper and lower branch solutions, in a certain range of the shrinking, mass suction, unsteadiness and curvature parameters. In order to establish which of these solutions are stable and which are not, a stability analysis has been performed. The effects of the governing parameters on the skin friction coefficient and dimensionless velocity profiles are presented graphically, and discussed in details. It results in that the curvature parameter affects considerably the flow characteristics. By our best of knowledge these results are new and original.

2. Basic equations

Consider the unsteady two-dimensional boundary layer flow of a viscous and incompressible fluid over a permeable curved stretching/shrinking surface coiled in a circle of radius R about the curvilinear coordinates r and s, as it is shown in Fig. 1(a), so that large values of R correspond to a slightly curved sheet. The geometry of the stretching sheet is shown in Fig. 1(b), while that for the shrinking sheet is shown in Fig. 1(c). However, there is no free stream velocity in both Fig. 1(b) and (c). It is assumed that the surface is stretched/shrinked with the velocity $u_w(s)$ along the s direction. It is also assumed that $v_w(t, s)$ is the mass flux velocity, where $v_w(t, s) < 0$ corresponds to suction and $v_w(t, s) > 0$ corresponds to injection, respectively, and t is the time.

Under these assumptions along with the boundary layer approximations, the governing equations for the flow are given by, see Sajid et al. [19] or Bejan [22],

$$\frac{\partial}{\partial r}\left[(r+R)\,v\right]+R\,\frac{\partial u}{\partial s}=0 \tag{1}$$

$$\frac{u^2}{r+R}=\frac{1}{\rho}\,\frac{\partial p}{\partial r} \tag{2}$$

$$\frac{\partial u}{\partial t}+v\,\frac{\partial u}{\partial r}+\frac{R\,u}{r+R}\,\frac{\partial u}{\partial s}+\frac{u\,v}{r+R} = -\frac{1}{\rho}\,\frac{R}{r+R}\,\frac{\partial p}{\partial s}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r+R}\,\frac{\partial u}{\partial r}-\frac{u}{(r+R)^2}\right) \tag{3}$$

where v and u are the velocity components along r- and s-directions, respectively, p is the pressure, ρ is the density and ν is the kinematic viscosity.

The appropriate initial and boundary conditions for the velocity components v and u are

$$\begin{aligned} &t<0: \quad v=u=0 \quad \text{for any } r \text{ and } s\\ &t\ge 0: \quad v=v_w, \qquad u=\lambda\,\frac{u_w(s)}{1-\alpha\,t}=\lambda\,\frac{a\,s}{1-\alpha\,t} \quad \text{at } r=0\\ &\qquad\quad u\to 0, \qquad \frac{\partial u}{\partial r}\to 0 \quad \text{as } r\to\infty \end{aligned} \tag{4}$$

where a is a positive constant, λ is the dimensionless constant stretching ($\lambda > 0$) or shrinking ($\lambda < 0$) parameter, respectively, and $\alpha > 0$ for an accelerated sheet and $\alpha < 0$ for a decelerated sheet, respectively and $u_w(s) = a\,s$.

3. Steady-state case

In order to solve Eqs. (1)–(3), we introduce the following similarity variables

$$u=\frac{a\,s}{1-\alpha\,t}\,f'(\eta), \qquad v=-\frac{R}{r+R}\sqrt{\frac{a\,\nu}{1-\alpha\,t}}\,f(\eta),$$
$$p=\frac{\rho\,a^2\,s^2}{(1-\alpha\,t)^2}\,P(\eta), \qquad \eta=\sqrt{\frac{a}{\nu\,(1-\alpha\,t)}}\,r \tag{5}$$