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Structural sensitivities of soft and steep nonlinear global modes in spatially developing media

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ABSTRACT

Structural sensitivities of soft and steep nonlinear global modes arising in a complex Ginzburg–Landau model are investigated by calculating the leading-order variation of their amplitude and frequency to an open-loop forcing and a closed-loop perturbation. The soft global mode is found to be sensitive to both the open-loop and closed perturbations in the region of linear absolute instability where its amplitude is not negligible. In particular, the frequency of the soft global mode exhibits large sensitivity at the location where the frequency of soft global mode was shown to be determined in the previous WKBJ analysis. On the other hand, the steep global mode shows a large response in the amplitude and the frequency to the open-loop perturbation located far upstream. To the closed-loop perturbation, the steep global mode is most sensitive at the location where the stationary front is located, consistent with the previous WKBJ analysis. Finally, the sensitivities analyzed for the fully nonlinear global mode are compared to those obtained from a weakly nonlinear analysis. It shows that the weakly nonlinear analysis fails to capture the sensitivity behavior obtained from the fully nonlinear global mode particularly under the strong advection yielding steep global mode.

1. Introduction

Since the introduction of the spatio-temporal analysis of instabilities in idealized parallel flows (i.e. absolute and convective instabilities) [1–4], the globally synchronized nonlinear structures appearing in open shear flows such as wake [5,6], mixing layer [7], and hot jet [8], have been understood in terms of the 'global' mode, which often refers to an eigenstructure resulting from a temporally growing instability over the entire spatial domain. Such a nonlinear structure is often tuned with a well-defined characteristic frequency and remains as a large-scale coherent motion even in fully-developed turbulent flows. Therefore, significant effort has been made for understanding the nature of such structures. The reader may refer to reviews by Huerre [9] and Chomaz [10] for further details on this issue.

Theoretical description of the linearly growing global mode has been well established. One may classify it into two categories: local and global approaches. The local approach is based on a WKBJ theory with the assumption that the base flow is weakly non-parallel (i.e. $X = \epsilon x$ where x denotes the streamwise direction with $\epsilon \ll 1$). In this circumstance, the evolution of a disturbance obeys the local dispersion relation obtained by setting the base flow at a given streamwise location X to be parallel. The frequency ω_{LG} of the linearly growing disturbance over the entire flow domain is then determined in terms of the local absolute frequency $\omega_0(X)$ [11–13]:

$$\omega_{LG} = \omega_0(X_l^s) \quad \text{with} \quad \left.\frac{\partial \omega_0}{\partial X}\right|_{X=X_l^s} = 0, \tag{1}$$

where X now turns out to be the complex streamwise location. It is important to note that the criterion implies that a finite region of local absolute instability ($\omega_{0,i}(X) > 0$) is necessary for $\omega_{LG,i} > 0$ [12,13], suggesting physical importance of the local absolute instability in generating linear global instability.

In many practical situations, the flow configuration, however, often involves strong non-parallelism and complexity. The linear global mode and its eigenfrequency ω_{LG} in this case are obtained by numerically solving the global eigenvalue problem. The earliest approach of this type was probably performed by Zebib [14] and Jackson [15] who successfully predicted the critical Reynolds number for the onset of Kármán vortex shedding in bluff-body wakes. The use of the adjoint global mode was introduced by Hill [16] for the control of linear global mode, whose work also led [17] with a similar approach for local absolute instability. Giannetti and Luchini [18] recently revisited the work by Hill [16]

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and performed a sensitivity analysis to identify the region where a small localized feedback forcing leads to a large drift of the linear global frequency. They showed that the linear global frequency is highly sensitive if the forcing is located in the region where the regular and adjoint global modes overlap. Importantly, this location is found to be very similar to the region where the presence of a small secondary cylinder stabilizes vortex shedding [19] as also demonstrated by Marquet et al. [20]. Recently, the computed regular and adjoint global modes have also been used by combining with the classical weakly nonlinear theory, and it has provided much deeper physical insight into the dynamics of global instability particularly when multiple linear global modes compete with each other for nonlinear pattern selection (see also e.g. Meliga et al. [21] among others).

Despite the encouraging progress, both the local and global approaches with linear global mode are, in principle, valid only in the regime where the role of nonlinearity is weak: i.e. the regime where the bifurcation control parameter such as the Reynolds number is not very far from the onset of the instability. Theoretical effort has therefore been made to describe the global mode in the 'fully' nonlinear regime where the bifurcation control parameter is sufficiently far from the onset. While the local approach with the assumption of parallel or weakly non-parallel flow was maintained, the dynamics of a nonlinear global mode were described in terms of the front propagating stable to unstable state [22–32]. In the fully nonlinear regime, a growing disturbance nucleates into a front, the downstream of which is composed of nonlinear instability waves. As the propagating velocity of the front is often identical to that of the leading and trailing edges of a linear wavepacket [33,34], the local absolute instability in the medium pushes the nucleated front to propagate. The front subsequently loses its velocity at the location where the nature of the local instability transits from convective to absolute ($X = X^{ca}$), and settles at this location while generating instability wave downstream. The stationary front therefore acts as a wavemaker, and the frequency of the nonlinear global mode ω_{NG} is given by local 'linear' absolute frequency at $X = X^{ca}$: i.e.,

$$\omega_{NG} = \omega_0(X^{ca}). \tag{2}$$

The nonlinear global mode, the frequency of which is given by (2), has been referred to as 'steep' or 'elephant' mode due to the sharp stationary front in its spatial structure (see also Fig. 2 and [35]). Application of the frequency selection criterion (2) to a number of canonical flows such as wakes [31,36], hot jets [8], and swirling vortex [37] has been shown to be successful.

By definition, the nonlinear global mode is just a nonlinear solution of the time-dependent governing equation, which is easily computed with well-established modern CFD solvers. Therefore, as in the case of linear global mode, one may think of a global approach which identifies the region where a certain type of forcing would yield a large drift of the amplitude and/or the frequency of a nonlinear global mode. The scope of the present study is to address this issue by designing a sensitivity analysis of a nonlinear global mode in the fully nonlinear regime. However, the sensitivity analysis for a nonlinear global mode has been very rarely carried out, and, to the best of my knowledge, only two works have addressed so far: [38,39]. The former computed sensitivity of the amplitude (energy) of the stationary nonlinear global mode in a real Ginzburg–Landau equation, whereas the latter calculated sensitivity of the frequency of the temporally periodic nonlinear global mode (Kármán vortex shedding) in a circular cylinder wake. Unfortunately, the two approaches have not been applied to the same flow configuration, thus it is difficult to see what kind of differences would be yielded depending on the objective functional of interest (i.e. amplitude or frequency). Furthermore, their results have not been well discussed in comparison with the physical insight gained from the previous local analyses using the framework of the front propagation. To fill these gaps, the present study is aimed at applying the two approaches to a supercritical complex Ginzburg–Landau (CGL) equation, which yields nonlinear global modes extensively analyzed by a nonlinear WKBJ analysis [28–30].

This paper is organized as follows. In Section 2, we briefly introduce linear and nonlinear global modes in the CGL equation with their frequency criterion. In particular, two types of nonlinear global mode respectively called 'soft' and 'steep' modes are illustrated [28–30]. In Section 3, structural sensitivities of the amplitude and the frequency of a nonlinear global mode to small open-loop and closed-loop perturbations are formulated following Hwang and Choi [38] and Luchini et al. [39]. The computed sensitivities are then shown in Section 4 with a discussion. In particular, we compare them with those obtained using a weakly nonlinear analysis. Finally, a summary and a few remarks are given in Section 5.

2. Nonlinear global modes in the complex Ginzburg–Landau equation

We consider a complex Ginzburg–Landau equation given in the following form [30]:

$$i\frac{\partial\psi}{\partial t} = \left(\omega_0(x) + \frac{1}{2}\omega_{kk}k_0^2\right)\psi + i\omega_{kk}k_0\frac{\partial\psi}{\partial x} - \frac{1}{2}\omega_{kk}\frac{\partial^2\psi}{\partial x^2} + \gamma|\psi|^2\psi, \tag{3a}$$

with boundary condition

$$\psi(x = \pm\infty) = 0. \tag{3b}$$

Here, $\psi(x, t)$ is the complex state function with the streamwise direction $x \in (-\infty, \infty)$ and the time $t \in [0, \infty)$. The parameters $k_0, \omega_0(x), \omega_{kk}$ and γ are set to be complex. We note that this form of the Ginzburg–Landau equation is convenient for the local stability analysis, as we shall see in Section 2.1 where the details of the parameters, k_0, ω_0, and ω_{kk} are given. For simplicity, we set k_0, ω_{kk}, and γ to be constant, while $\omega_0(x)$ is considered to vary along x such that:

$$\omega_0(x) = \omega_0^{max} + \frac{1}{2}\omega_{0xx}x^2, \tag{4}$$

where ω_{0xx} is a complex constant. It should be mentioned that $\omega_0(x)$ here is purposely set as in (4) to avoid the term depending linearly on x (see also Section 2.2.2 for further discussion).

In setting the coefficients of (3), there are some requirements to ensure the well-posedness of (3). It is useful to rearrange (3) as

$$\frac{\partial\psi}{\partial t} + U\frac{\partial\psi}{\partial x} = \mu(x)\psi + D\frac{\partial^2\psi}{\partial x^2} - i\gamma|\psi|^2\psi, \tag{5a}$$

where

$$U \equiv -\omega_{kk}k_0, \qquad \mu(x) \equiv -i\left(\omega_0(x) + \frac{1}{2}\omega_{kk}k_0^2\right), \qquad D \equiv \frac{i}{2}\omega_{kk}. \tag{5b}$$

This form suggests that U can be interpreted as the complex advection velocity, $\mu(x)$ a function controlling local instability property, and D the complex diffusivity where its imaginary part acts for dispersion. First, to restrict our discussion only to the case of downstream advection, it is necessary to set $U_r \geq 0$, giving $k_{0,i} \leq 0$. Also, the well-posedness of (3) with its boundary condition ($\psi(x = \pm\infty) = 0$) would require any perturbations in ψ to decay at $x = \pm\infty$, yielding $\omega_{0xx,i} < 0$. The positive diffusivity ($D_r > 0$), which also ensures the causality of (3), gives $\omega_{kk,i} < 0$. Finally, to obtain stably sustaining nonlinear global modes, the nonlinear term should be stabilizing, requiring $\gamma_i < 0$.