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On the transition to turbulence of wall-bounded flows in general, and plane Couette flow in particular

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HIGHLIGHTS

- Reviews recent achievements about transition to turbulence in wall-bounded flows.
- Highlights laminar–turbulent oblique band coexistence in plane Couette flow.
- Analyzes respective relevance of temporal chaos and spatiotemporal chaos.
- Introduces account of transition using the tools of statistical physics.
- Presents new results and a model for band patterning in Couette flow.

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ABSTRACT

The main part of this contribution to the special issue of EJM-B/Fluids dedicated to Patrick Huerre outlines the problem of the subcritical transition to turbulence in wall-bounded flows in its historical perspective with emphasis on plane Couette flow, the flow generated between counter-translating parallel planes. Subcritical here means discontinuous and direct, with strong hysteresis. This is due to the existence of nontrivial flow regimes between the *global stability* threshold Re_g, the upper bound for unconditional return to the base flow, and the *linear instability* threshold Re_c characterized by unconditional departure from the base flow.

The *transitional range* around Re_g is first discussed from an empirical viewpoint (Section 1). The recent determination of Re_g for pipe flow by Avila et al. (2011) is recalled. Plane Couette flow is next examined. In laboratory conditions, its transitional range displays an oblique pattern made of alternately laminar and turbulent bands, up to a third threshold Re_t beyond which turbulence is uniform.

Our current theoretical understanding of the problem is next reviewed (Section 2): linear theory and non-normal amplification of perturbations; nonlinear approaches and dynamical systems, basin boundaries and chaotic transients in minimal flow units; spatiotemporal chaos in extended systems and the use of concepts from statistical physics, spatiotemporal intermittency and directed percolation, large deviations and extreme values. Two appendices present some recent personal results obtained in plane Couette flow about patterning from numerical simulations and modeling attempts.

Cautionary note about the literature cited. The number of articles related to the topics examined here is tremendous and, though already referring to more than 150 publications, the bibliography is far from exhaustive by at least one order of magnitude. For a better coverage, the reader is invited to consult the literature cited in the review papers mentioned. I tried not to bias the list according to my personal interests, while choosing what I thought to be the most representative papers in each subtopic, sometimes the most recent publications of given people or groups reviewing related works in their introductions and pointing backwards to earlier relevant papers. For convenience, references are listed in alphabetical order of the first author and next chronologically.

The article published by O. Reynolds in 1883 [130] founded the scientific approach to the problem of the transition to turbulence. Already an abstract in itself, its title "*An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous and the law of resistance in parallel channels*," summarized the main features of the problem and, between the words, identified its control parameter Re nowadays called the Reynolds number. This parameter is a measure of the typical shear

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present in the flow under consideration.[1] When Re is small viscous effects have enough time to tame departures from the base flow profile so that 'direct motion' in Reynolds' own terms, i.e. *laminar* flow, prevails. On the contrary, when Re is large 'sinuous motion' can be amplified up to being considered as *turbulent*. The problem is then to determine/predict the value of Re at which the transition takes place.

On general grounds two characteristic values can be defined [79,99,70], a threshold for *unconditional* or *global stability* $\mathrm{Re_g}$, and a threshold for *unconditional instability* $\mathrm{Re_c}$, 'c' for 'critical'. Thresholds $\mathrm{Re_g}$ and $\mathrm{Re_c}$ are *global* and *local* quantities, respectively. These terms have to be understood in the general context of dynamical systems: In the state space, 'global' means whatever the amplitude and shape of the perturbation brought to the base state, whereas 'local' means infinitesimal, which allows linearization and eigenmode decomposition. $\mathrm{Re_c}$ is obtained from linear stability analysis that can be continued in the weakly nonlinear regime around threshold by perturbation. At this level, issues are in principle purely technical (but possibly delicate) in a well-posed setting.

Obviously, $\mathrm{Re_g}$ lies below $\mathrm{Re_c}$ and between $\mathrm{Re_g}$ and $\mathrm{Re_c}$ stability is only *conditional*: it depends on the shape and intensity of perturbations brought to the base flow. Global stability is therefore much difficult to ascertain since the variety of possible perturbations cannot be tested in any systematic way. In a few cases, one can show that local and global thresholds coincide, which makes the transition *supercritical*. Thermal convection in a horizontal layer heated from below is the most celebrated example of such a circumstance [117]. This case is exceptional and, in general, permanent departures from the base state may exist in the *subcritical range* below $\mathrm{Re_c}$. The *energy method* [79,117] generates a lower bound $\mathrm{Re_E}$ to the global stability threshold. $\mathrm{Re_E}$ is the threshold below which the kinetic energy contained in any perturbation to the base flow decreases to zero in a monotonic way. This bound is usually very conservative. By contrast, the condition defining $\mathrm{Re_g}$ bears on the ultimate decay of the perturbations, possibly at the end of long transients during which the energy may vary wildly before decreasing like below $\mathrm{Re_E}$.

Linear instability deals with infinitesimal perturbations that can be analyzed as superpositions of elementary modes of infinite spatial extension, e.g. Fourier modes. *A contrario*, typical perturbations living below $\mathrm{Re_c}$ have finite amplitudes and finite supports, and coexist with laminar flow. These are the *flashes of turbulence* observed by Reynolds in his pipe or the *turbulent spots* seen in planar geometries. When the applied shear is very large, the system is expected to be uniformly turbulent. At least conceptually, one should therefore find another threshold separating laminar–turbulent coexistence from uniform turbulence since this represents two qualitatively different situations. The localization of such a threshold, called $\mathrm{Re_t}$ in the following, will also be discussed below. What is generally called the *transitional range* is therefore the Reynolds number interval extending from around $\mathrm{Re_g}$ to around $\mathrm{Re_t}$. Table 1 recapitulates known values of these thresholds for the two cases of main interest here, pipe flow and simple shear flow, both of them with $\mathrm{Re_c} = \infty$.

Our understanding of the *transitional range* in wall-bounded flows has made considerable progress recently. Relevant information can be found in the proceedings of the 2005 IUTAM Symposium edited by Mullin & Kerswell [112]. In 2008, a whole issue of the Philosophical Transactions of the Royal Society has been devoted to the celebration of the 125th anniversary of the publication of Reynolds' article, where discussions of experimental and theoretical findings for pipe flow, also known as *Hagen–Poiseuille flow* (HPF) can be found [52]. Several reviews have also appeared, focusing on theoretical and numerical aspects [51,170] or on the experiments [170,113], summarizing the state of the art before 2010. Accordingly, in Section 1.1 I shall limit myself to a brief account of posterior results in HPF centered on the quantitative determination of $\mathrm{Re_g}$ [3] that will be defined as the value of Re below which the flashes of turbulence always decay in the long term and above which they are able to split and spread turbulence in the pipe.

Table 1
Characteristic values of the control parameter in some wall-bounded flows. The ingredients for the Reynolds number as introduced in note 1, $\mathrm{Re} = UL/\nu$, are the mean speed U and the diameter of the pipe D for Hagen–Poiseuille flow (HPF); for plane Poiseuille flow (PPF) and plane Couette flow (PCF) L, usually noted h, is the 1/2-distance between the plates; for PPF U is the speed of the laminar flow in the center plane and for PCF the speed of the driving plates. $\mathrm{Re_g}$ is the global stability threshold, $\mathrm{Re_c}$ the linear stability threshold, and $\mathrm{Re_t}$ the threshold beyond which turbulence is featureless.

Flow	$\mathrm{Re_E}$ [79]	$\mathrm{Re_g}$	$\mathrm{Re_c}$	$\mathrm{Re_t}$
HPF	81.5	2040 [3]	∞ [135]	$\sim$2700 [171]
PCF	20.7	$\sim$325 [17]	∞ [132]	$\lesssim$ 415 [124], Appendix
PPF	49.6	$\sim$840 [160]	5772 [118]	$\gtrsim$ 1600 [158]

With respect to simple shear flow, also called *plane Couette flow* (PCF), only partial reviews of experimental and numerical results seemingly exist [126,51]. I shall not attempt to be comprehensive but try to focus on features that, in my opinion, are the most interesting. Accordingly, in Section 1.2 I will just sketch the history of the subject and present experimental results gathered by the Saclay group [126] in the perspective of earlier and more recent numerical findings. In a first series of experiments by this group, concluded with Bottin's thesis [16], the focus was on the identification of mechanisms and the determination of $\mathrm{Re_g}$ based on the dynamics of turbulent spots in setups with moderate aspect ratio.[2] Later, in the larger aspect ratio setup used by Prigent [124], patterns of alternately laminar and turbulent oblique bands were shown to occupy most of the transitional range, leading to the determination of the upper threshold $\mathrm{Re_t}$. I shall situate these findings in their context and relate them to the transition in cylindrical Couette flow (CCF) which has PCF as its small gap limit and the banded regime as the limit of *spiral turbulence* observed in that system [27,1,124].

Other cases of comparable interest, especially in view of applications, will not be reviewed here, in particular *plane Poiseuille flow* (PPF), the flow between two motionless plates driven by a pressure gradient, and the Blasius boundary layer flow [136]. Both are linearly unstable above some finite critical Reynolds number $\mathrm{Re_c}$ but also display nontrivial subcritical flow in the form of turbulent spots promoting developed turbulence when the level of residual turbulence in the base flow is large (natural transition) or when it is clean enough but appropriately triggered. Thresholds for PPF are also quoted in Table 1.

Some views on the present theoretical understanding of the transition will next be presented in Section 2. I shall first recall linear properties related to the stability of wall-bounded flows compared to free shear flows [77,137] and the importance of non-normal energy growth [156,138], streamwise vortices and *lift-up* [54,88], and the process underlying the sustainment of flow patterns away from laminar flow uncovered by Waleffe and

[1] Explicitly, $\mathrm{Re} = UL/\nu$, where U is a typical amplitude of velocity variations, L a typical distance over which the speed varies, and ν the kinematic viscosity of the fluid. It can be understood as the ratio of a viscous time scale $\tau_\mathrm{v} = L^2/\nu$ to an advection time scale $\tau_\mathrm{a} = L/U$.

[2] For PCF, two aspect ratios can be defined, $\Gamma_x = L_x/2h$ and $\Gamma_z = L_z/2h$ where L_x and L_z are the streamwise and spanwise dimensions of the shearing zone, and $2h$ the gap between the moving plates. For HPF, this would just be L/D where L is the length of the pipe and D its diameter.