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The effect of rounding corners or cutting edges on the absolute/convective instability properties of mixing layers

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ABSTRACT

We revisit the inviscid spatio-temporal stability properties of mixing layers. Earlier comparisons of the dispersion relations pertaining to the broken line model and the hyperbolic tangent profile have shown that the temporal stability properties are mainly governed by the velocity difference and the shear layer thickness, and that they are very robust to the details of the velocity profile. The situation however dramatically changes when one considers the spatio-temporal stability properties of these two limiting cases, which were shown to differ significantly. With the aim to better understand these strong differences, we introduce a family of velocity profiles that continuously spans from the broken line model to the tanh profile. We show that the momentum thickness appears as an important parameter that helps better predicting the absolute/convective properties of a given mixing layer.

1. Introduction

The concept of absolute and convective instability was first applied to the analysis of mixing layers in the landmark paper of Huerre and Monkewitz [1]. Mixing layers are primarily characterized by three non-dimensional parameters: the Reynolds number, the shear layer thickness and the advection parameter (or velocity ratio) R that compares the velocity difference between the streams to their mean value. Using the tanh velocity profile first introduced by Michalke [2], Huerre and Monkewitz [1] found that a finite amount of counterflow was required for the flow to become absolutely unstable, which could be quantified in the transition value of the advection parameter $R^* = 1.315$. This prediction was later remarkably confirmed experimentally by Strykowski and Niccum [3].

It is a common belief that the temporal stability properties of a shear layer can be easily determined by using the broken-line velocity profile, first introduced by Lord Rayleigh [4], provided the advection parameter and shear layer thickness are known. Indeed, the temporal stability properties, while not being exactly similar, share common features: both follow the same slope at $k = 0$, given by the Kelvin–Helmholtz vortex layer instability. Both also display a rounded bell shape with a maximum growth-rate of approx 0.2 attained for a wavenumber that scales like the cut-off wavenumber and as the inverse of the shear-layer thickness. This is natural since this is the only length-scale in this problem. As stated by Villermaux [5], "the essence of the problem is fully contained in this caricature which presents the great advantage [...] of a transparent analytical result".

The broken-line velocity profile has not only become popular because it led to an analytical dispersion relation, but also because the presence of edges in the velocity profile, or in other words of vorticity sheets, naturally introduces an edge-wave or Rossby-wave perspective [6], that allowed for instance new interpretations of spatio-temporal stability properties of confined wakes [7]. The robustness of the dispersion relation with respect to a regularization of the corner discontinuity has been assessed by Balsa [8].

However, when it comes to determine the velocity ratio at the transition between absolute and convective, R^*, the broken line model and the continuous profiles start to behave significantly differently. As analysed in detail by Ortiz, Chomaz and Loiseleux [9], the saddle point pinches at infinity as the critical advection parameter $R^* = 1$ is reached. This suggests that the details of the velocity profile do matter and that comparison in experiments can only be drawn if the experimental flow fields are measured with sufficient accuracy, as also concluded by Marquet and Lesshafft [10], who have pointed out the large sensitivity of the spatio-temporal stability properties with respect to minute changes in the velocity profile.

The purpose of the present study is to analyse the continuous transition from the broken-line shear layer profile to the continuous tanh profile. The erf solution being an asymptotic solution of

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Table 1
Main stability characteristics of the broken line mixing layer model, the erf profile and the tanh profile: maximum growth-rate σ_{max}, associated wavenumber k_{max}, cut-off wavenumber k_c, critical velocity ratio for absolute/convective instability transition R^* and momentum thickness Θ.

	σ_{max}	k_{max}	k_c	R^*	Θ
broken line	0.2	0.2	0.32	1	1/6
erf profile	0.19	0.21	0.45	1.28	$1/\sqrt{2\pi}$
tanh profile	0.19	0.22	0.5	1.315	1/4

the Navier–Stokes equations in the large Re and small R limits [11] is also considered. We ask the question whether the spatio-temporal properties continuously depend on the degree of regularization of the corner discontinuity or if the broken line model is a singular limit.

2. Governing equations

We consider a mixing layer $U(y)$ bridging two streams of respective parallel velocities U_1 and U_2 over a distance characterized by the vorticity thickness

$$\delta_\omega = \frac{|\Delta U|}{\max(|\Omega(y)|)}, \tag{1}$$

where $\Delta U = U_2 - U_1$ and $\Omega(y) = -\frac{dU}{dy}$ is the transverse vorticity. Introducing the mean velocity of the stream $\bar{U} = (U_1 + U_2)/2$, the velocity profile can be written as

$$U(y) = \bar{U} + \frac{\Delta U}{2} u\left(\frac{4y}{\delta}\right), \tag{2}$$

where u is an odd increasing function asymptoting 1 at $+\infty$ with prescribed slope in 0 ($\frac{du}{dy}(0) = \frac{1}{2}$), which entirely characterizes the shape of the profile. Upon introducing $\bar{U}$ as velocity scale and $\delta_\omega/4$ as length scale, the velocity profile reduces to

$$U(y) = 1 + Ru(y), \tag{3}$$

where the notation has not been changed to account for non-dimensional variables. The parameter $R = (U_2 - U_1)/(U_1 + U_2)$ is a measure of the velocity difference across the layer and is referred to as advection parameter or velocity ratio. If $0 < R < 1$ both streams run in the same direction, while for $R > 1$ they flow in opposite directions. In the limiting case where $R = 0$ there is no shear, and, when $R = 1$, only one stream is present.

Considering the inviscid limit, we shall assume that the spatial evolution of the mixing layer is so small over a wavelength of the instability wave that the mean flow may be considered as parallel. Under these conditions, a small disturbance of wavenumber k and frequency ω, is defined by the perturbation stream function

$$\Psi(x, y, t) = \psi(y)\exp(i(kx - \omega t)). \tag{4}$$

As derived in classical textbooks [12], the eigenfunction $\psi(y)$ then satisfies the Rayleigh equation

$$[U(y) - \omega/k](\psi'' - k^2\psi) - U''(y)\psi = 0, \tag{5}$$

with boundary conditions $\psi(\pm\infty) = 0$. We discretize the above Rayleigh equation using a pseudo-spectral collocation method including an algebraic mapping [13] using $N = 80$ collocation points. The discrete eigenvalue problem is solved using matlab.

3. Base flows

The most studied velocity profile modelling a mixing layer is the tanh profile as popularized by Michalke [2],

$$u_{\tanh}(y) = \tanh(y/2). \tag{6}$$

However, as detailed in Monkewitz and Huerre [11], the profile emerging naturally from a boundary layer type asymptotic expansion at large Reynolds numbers and small values of the shear parameter R is the error function profile.

$$u_{\mathrm{erf}}(y) = \mathrm{erf}\,(\sqrt{\pi}y/4). \tag{7}$$

Finally, the broken line velocity profile introduced by Rayleigh [14] is defined by

$$\begin{aligned} u_{\mathrm{broken}}(y) &= -1 \quad \text{when } y < -2 \\ u_{\mathrm{broken}}(y) &= y/2 \quad \text{when } -2 < y < 2 \\ u_{\mathrm{broken}}(y) &= 1 \quad \text{when } y > 2. \end{aligned} \tag{8}$$

These three profiles share the same shear layer thickness. In order to quantify the differences between these profiles, it is natural to use their momentum thickness, defined as

$$\Theta = \int_{-\infty}^{\infty} (1 - u(y))(1 + u(y))dy. \tag{9}$$

The momentum thickness (made non-dimensional by $4\delta_\omega$) associated to the three reference profiles are reported in Table 1. They are seen to increase as the edges are progressively cut from $\frac{1}{6}$ for the broken line model to $\frac{1}{4}$ for the tanh profile, through the value $\frac{1}{\sqrt{2\pi}}$. The effect of rounding the corners of the broken line profile is to increase the momentum thickness, in accordance with its definition associated to the momentum deficit of the velocity profile, as compared to the vortex sheet model jumping abruptly from one stream velocity to the other at $y = 0$.

It is instructive to compare the corresponding vorticity distributions to these velocity profiles and in particular of the two limiting velocity profiles considered (tanh and broken line). For the broken line profile, the associated vorticity distribution is a square window (or top hat) function while it is a Gaussian vorticity distribution for the erf profile and a sech^2 distribution for the tanh.

$$\Omega_{\tanh}(y) = \mathrm{sech}^2(y/2)/2, \tag{10}$$

$$\Omega_{\mathrm{erf}}(y) = \exp(-\pi y^2/16)/2, \tag{11}$$

$$\begin{aligned} \Omega_{\mathrm{broken}}(y) &= 0 \quad \text{when } y < -2 \\ \Omega_{\mathrm{broken}}(y) &= 1/2 \quad \text{when } -2 < y < 2 \\ \Omega_{\mathrm{broken}}(y) &= 0 \quad \text{when } y > 2. \end{aligned} \tag{12}$$

Observe that all vorticity distributions have the same maximum (equal to 1/2 in non dimensional units) and the same vorticity integral.

In order to continuously interpolate through these three profiles, we introduce a family of profiles that results from the solutions of the diffusion equation with initial condition the top hat profile or the sech^2 profile and in both cases the Gaussian distribution as asymptotic solution:

$$\frac{\partial \Omega}{\partial \tau} = \frac{\partial^2 \Omega}{\partial y^2}, \tag{13}$$

$$\Omega(\pm\infty, \tau) = 0, \tag{14}$$

$$\Omega(y, \tau = 0) = \Omega_{\tanh}(y) \quad \text{or} \quad \Omega_{\mathrm{broken}}(y). \tag{15}$$

At each time step the vorticity distribution is rescaled to keep a constant integral and constant maximum. The associated velocity profiles, retrieved by numerical spatial integration are plotted together with the vorticity distributions in Fig. 2(a), (b). Since the time τ appears here only as an artificial parameter, the above profiles are now labelled by their momentum thickness Θ.

4. Temporal stability

Let us first consider the temporal stability framework where the wavenumber of the instability is prescribed $k \in \mathbb{R}$ and the complex