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Global stability analysis of underexpanded screeching jets

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ABSTRACT

This article deals with a global stability analysis of the screech phenomenon. We have shown that a laminar underexpanded supersonic cold jet can exhibit globally unstable modes. A closer look at the structure of these modes shows that they present upstream propagating waves, which is known to be a major component of the screech phenomenon. Furthermore, we find a good agreement between the frequency of the eigenmodes and existing empirical formulas for the prediction of screech frequency. We have then studied the influence of two key parameters on the linear stability of the flow, the jet pressure ratio (JPR) and the nozzle lip thickness, which are known to play an important role in the screech phenomenon. Finally, a careful study of the structure of the unstable modes shows that the upstream propagating acoustic waves of those modes are generated by supersonic phase velocity disturbances, a well-known sound generation mechanism.

1. Introduction

The study of imperfectly expanded supersonic jets is an active subject of research since such flows can be found in a broad variety of industrial applications. The most common one is military aircrafts, whose engines often operate at off-design conditions. In this article, we focus on underexpanded jets, where the flow pressure at the nozzle exit is higher than the ambient pressure. This mismatch in pressure induces the apparition of a complex quasi-periodic "shock-cell" structure. The jet periodically overexpands and reconverges, attempting to match the ambient pressure, and consequently, forms a standing wave pattern. As a result, shocks and expansion fans appear periodically, creating the so-called shock-cells. Despite the fact that those flows are highly nonlinear, it is possible to predict the gross features of such jets, such as the shock-cell length, with a good agreement with experimental data [1–3].

One of the important features of supersonic jets is that they can generate strong noise, a point which has been intensively studied in the past decades. A large number of articles on the subject have been written since the first work of Lighthill in 1952 [4]. One may refer for instance to the review of Tam [5] for further details. It is now known that the noise of shock-containing supersonic jets has three components: the broadband shock-associated noise resulting from the interaction of instability waves and the shocks, the turbulent mixing noise generated by the turbulent fluctuations, and the screech tones, which are the subject of this article. More information about the two first noise components can be found in Tam 1995 [5].

The screech phenomenon was first studied in 1952 by Powell [6]. He observed that, under certain conditions, supersonic imperfectly expanded jets can produce very loud discrete frequency tones, the so-called screech tones. This phenomenon can be so intense that in real flight conditions, it can damage the structure of an aircraft. The first observation of such damages was made by the British Aircraft Corporation in the 1960s, where in-flight measurements showed that screech was responsible for minor cracking on VC 10 aircrafts [7,8]. Such concerns do not affect most of modern commercial engines though, and usually, screech tones are observed only with military aircrafts.

In one of his papers, Tam [5] refers to screech as "the least understood, least predictable component of supersonic jet noise". Indeed, many questions, such as the prediction of the amplitude of the noise, or its sensitivity to the surrounding environment, have remained unanswered. However, the dominant physical mechanism is known and has been described by Powell [6] as a feedback loop between the shocks and the nozzle lip: instability waves developing in the shear layer interact with the shocks, giving birth to acoustic waves propagating upstream. When those waves reach the nozzle lip, they are reflected and excite the shear layer, giving birth to new embryo perturbations that undergo the same process, closing the resonant loop.

There is abundant literature available on the topic of screeching jets: as mentioned in Raman 1999 [9], from Powell's first

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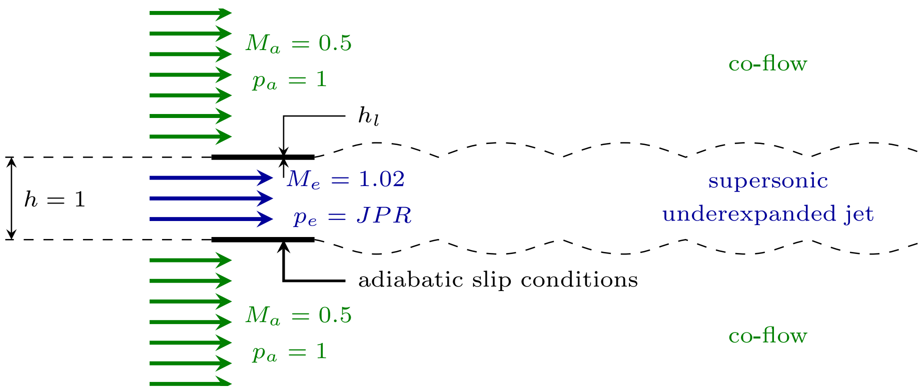

Fig. 1. Scheme of the physical configuration of the flow (nondimensional quantities). The two variable parameters are the JPR and the lip thickness h_l.

observation to now, more than 200 papers have been published. An extensive bibliography and a detailed review on screech can be found in the article of Raman [8]. But despite the large amount of studies that can be found on the topic, our knowledge of the phenomenon remains mainly qualitative. The only real quantitative prediction available is the frequency of the tones [6]. This lack of understanding is the reason screech is still an active field of research. But, to our knowledge, screech has never been studied in the light of a linear global stability analysis, which is the purpose of this article.

Recent works have shown that stability theory appears as a very successful framework for sound prediction in jets. We can cite for example the article of Lesshafft [10] on global modes in adapted subsonic hot jets, the work of Ray and Lele [11] on broadband shock-associated noise in supersonic underexpanded jets using parabolized stability equations (PSE), or Nichols [12], who performed a global mode decomposition on supersonic adapted jets. The case of screech presents one strong particularity: as briefly explained above, one of the key features of a screeching jet is that it presents upstream propagating acoustic waves that play a major role in the instability process. Consequently, a stability analysis based on PSE is unable to capture such upstream propagating structures and cannot therefore be used here. On the contrary, a global stability analysis [13,14], in which both the cross-stream and stream-wise directions of the perturbation are solved for, is able to capture upstream-propagating waves and may also handle more precisely the non-parallelism induced by the shock-cell structures. In the present article, we aim at finding an underexpanded supersonic jet that is globally unstable, and analyze the link between the unstable structures and the screech phenomenon.

The outline of the paper is as follows. After a brief reminder on global linear stability theory and on the numerical strategy adopted to perform the study, we will present an underexpanded jet configuration that is marginally unstable, and relate the features of the unstable global mode to screech. Then, we will assess the effects of two key parameters, the jet pressure ratio (JPR) and the lip thickness. In the last section, we will focus on the noise generation mechanism associated with the unstable global modes.

2. Physical configuration and linear global stability analysis

2.1. Physical configuration and governing equations

We focus on two-dimensional cold jets of air surrounded by a co-flow. In this study, the jet pressure ratio (JPR), defined as the ratio between the static jet pressure and the ambient pressure, and the lip thickness are the two parameters that will be varied. All other parameters of the configuration are fixed: the Mach number of the co-flow is 0.5, while the Mach number of the jet is 1.02. The height of the nozzle exit is equal to 3 mm. The stagnation temperature of both the jet and the co-flow is $T_0 = 288$ K, the Prandtl number is 0.72 and the viscosity follows a Sutherland law, with standard coefficients for air. The static pressure of the ambient air is set to 3000 Pa. The static pressure of the jet, and thus the Reynolds number (based on the jet velocity, the height of the nozzle, and the static density/temperature of the jet) depend on the JPR. The value of the height of the nozzle and the static pressure of the ambient air have been chosen such that, for all the studied JPR in this paper, the order of magnitude of the Reynolds number is 10^3, ensuring that the flow is in a laminar transitional situation. To simplify the study, we have imposed adiabatic slip conditions on the walls of the nozzle, so that the boundary layer thicknesses (inside and outside the nozzle) are zero at the nozzle exit: the effect of the boundary layer thickness is left for future work.

The flow dynamics is modeled using the compressible 2D Navier–Stokes equations, that can be recast in the following compact form:

$$\frac{d\mathbf{q}}{dt} = \mathcal{R}(\mathbf{q}), \tag{1}$$

where $\mathbf{q} = (\rho, \rho u, \rho v, \rho E)^T$ designates the variables describing the flow (density, streamwise momentum, cross-stream momentum, total energy) and $\mathcal{R}(\mathbf{q})$ designates the conservation of mass, momentum, and energy equations. From now on, we consider that all quantities are made nondimensional using the jet velocity, the height of the nozzle, the static pressure and density of the ambient air (see Fig. 1).

2.2. Global mode decomposition

A baseflow $\mathbf{q}_b$ is defined such that it is a stationary solution of Eq. (1). Therefore, we have $\mathcal{R}(\mathbf{q}_b) = \mathbf{0}$. If we consider a small perturbation $\mathbf{q}'$ around the baseflow, $\mathbf{q} = \mathbf{q}_b + \mathbf{q}'$, the linearization of (1) yields the following governing equation for the perturbation:

$$\frac{d\mathbf{q}'}{dt} = \mathcal{A}\mathbf{q}', \tag{2}$$

with $\mathcal{A} = \left.\frac{\partial \mathcal{R}}{\partial \mathbf{q}}\right|_{\mathbf{q}_b}$, the linearization of the operator $\mathcal{R}$ around the baseflow.

A global mode decomposition consists in finding particular solutions of (2) under the form

$$\mathbf{q}'(x, y, t) = e^{\lambda t}\hat{\mathbf{q}}(x, y), \tag{3}$$

where $\hat{\mathbf{q}}$ are the so-called global modes of the system and $\lambda = \sigma + i\omega$ is a complex scalar describing the time-behavior of the structure (σ is the amplification rate and ω the frequency of the mode). They describe the asymptotic behavior of the flow with

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