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Pairing of two vertical columnar vortices in a stratified fluid

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ABSTRACT

We present three-dimensional (3D) numerical simulations of the pairing of two vertical columnar vortices in a stably stratified fluid. Whereas in two dimensions, merging of two isolated vortices occurs on a diffusion time scale, in the three-dimensional stratified case we show that merging is a much faster process that occurs over an inertial time scale. The sequence of dynamical processes that leads to this accelerated pairing involves first a linear stage where the zigzag instability develops displacing vortices alternately closer and farther with a vertical periodicity scaling on the buoyancy length scale $L_B = F_h b$, where F_h is the horizontal Froude number ($F_h = \Gamma/\pi a^2 N$ with a the core size of the vortices, Γ their circulation and N the Brunt–Väisälä frequency) and b is the separation distance between the vortices. In layers where the vortices have started to move closer, their distance decreases exponentially with the growth rate of the zigzag instability. Non-linearities do not seem to affect this process and the decrease only stops when the pairing is completed in that layer. At the same time, enstrophy that has also grown exponentially reaches a magnitude of the order of the Reynolds number $Re = \Gamma/(\pi\nu)$ (where ν is the kinematic viscosity of the fluid) if the Reynolds number is not too large, meaning that energy is then dissipated on the inertial time scale. This dissipation occurs in thin layers and the vortices that were originally moving away in the intermediate layer start slowing down and rapidly merge.

1. Introduction

Atmosphere, oceans and some astrophysical fluids are stably stratified (see [1] for a review) and rotating. At mesoscale for the Earth's atmosphere, i.e. between 1 and 100 km, the planetary rotation is weak and the stratification controls the dynamics. Nastrom, Gage and Jasperson [2] reported that the kinetic energy spectrum versus horizontal wavenumber k_h is of the form $k_h^{-5/3}$ for the atmosphere in the mesoscale range, whereas it is of the form k_h^{-3} at larger scales. Following Lilly [3], they suggested that this $k_h^{-5/3}$ spectrum might be due to an inverse energy cascade from small (∼1 km) to large (∼500 km) scales, similar to the energy cascade predicted for two-dimensional (2D) turbulence by Kraichnan [4] and well confirmed by numerical simulations and experiments. In 2D, energy is transferred by the merging of two vortices to form a larger one [5]. The idea that the potential vorticity in a stratified flow even at slow time scale in the absence of the gravity wave component might behave as a 2D fluid was questioned by many authors [6–10]. In particular, [11,12] showed that several 2D flows were unstable when the fluid is stratified, and they named this instability the zigzag instability. Specifically, the zigzag instability affects co-rotating vortex pairs [13,12] and has a growth rate which scales as twice the external strain field generated by one vortex on the other ($S = \Gamma/2\pi b^2$). Thus, this instability is as fast as the rotation $\Omega = \Gamma/\pi b^2$ of the vortex pair. Destabilization should thus occur in a few rotations of the pair and as a consequence, this instability should strongly affect the merging between vortices, and may therefore help explaining the departure of the stratified turbulence from two-dimensional turbulence.

In the present paper, we investigate through numerical simulations a single pairing event in a strongly stratified fluid in order to find out to which extent stratification affects this

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process. In particular, we will compare this stratified merging to purely two-dimensional merging. The second section presents the numerical method used to study a pairing event by direct numerical simulations. The third section shows the qualitative behavior of the merging. The fourth, fifth and sixth sections describe and analyze in detail the pairing in a stratified fluid.

2. Numerical simulations

2.1. Governing equations and numerical method

The dynamics of the flow is governed by the incompressible Navier–Stokes equations under the Boussinesq approximation:

$$\frac{\partial \boldsymbol{u}}{\partial t} = \boldsymbol{u} \wedge \boldsymbol{\omega} - \nabla \left[p + \frac{\boldsymbol{u}^2}{2} \right] - \rho \boldsymbol{e}_z + \nu \Delta \boldsymbol{u}, \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \boldsymbol{u} \cdot \nabla \rho = N^2 u_z + D \Delta \rho \tag{2}$$

where $\boldsymbol{e}_z$ is the unit vector in the z-direction pointing upward, p the pressure field, $\boldsymbol{u}$ the non-divergent velocity ($\mathrm{div}\,\boldsymbol{u} = 0$), $u_z = \boldsymbol{e}_z \cdot \boldsymbol{u}$ its vertical component, D the diffusivity of the stratifying agent and ν the kinematic viscosity. The density field is the sum of a constant density ρ_0, a linear profile $\bar{\rho}(z)$ and a perturbation $\rho_0 \rho / g$. The density perturbation is rescaled by g/ρ_0 in order to avoid an extra constant in Eq. (1). The Brunt–Väisälä frequency is $N = \sqrt{-(g/\rho_0) d\bar{\rho}/dz}$, where g is the gravity acceleration.

Eqs. (1)–(2) are expressed in the Fourier space:

$$\frac{d\hat{\boldsymbol{u}}}{dt} = P(\boldsymbol{k}) \left[\widehat{\boldsymbol{u} \wedge \boldsymbol{\omega}} - \hat{\rho} \boldsymbol{e}_z \right] - \nu \boldsymbol{k}^2 \hat{\boldsymbol{u}}, \tag{3}$$

$$\frac{d\hat{\rho}}{dt} = N^2 \hat{u}_z - D \boldsymbol{k}^2 \hat{\rho}, \tag{4}$$

where the Fourier transform is denoted by a hat, $\boldsymbol{k}$ is the wavenumber and $P(\boldsymbol{k})$ is the projection operator on the solenoidal space. To compute (3)–(4), we use a pseudo-spectral solver adapted from the unstratified code used by [14]. The computational domain is a parallelepipedic box of height L_z with a square horizontal base ($L_x = L_y$ where L_x and L_y are the dimensions respectively in the x and y directions). The spatial resolution is chosen to be about the same in all directions implying that the numbers of collocation points on the horizontal directions are equal, $n_x = n_y$, and that the number of collocation points on the vertical is $n_z \sim n_x L_z / L_x$. Time integration is performed with a second order Adams–Bashforth scheme. Dissipative terms are integrated exactly. The 2/3 rule is applied for de-aliasing.

2.2. Initial conditions

The initial velocity field $\boldsymbol{U}$ is made of a quasi-steady 2D pair of co-rotating vortices $\boldsymbol{U}_{2D}(x, y)$ perturbed by the most unstable 3D eigenmode $\boldsymbol{U}'$

$$\boldsymbol{U}(x, y, z, t = 0) = \boldsymbol{U}_{2D}(x, y) + A\, \Re(e^{ik_{zm}z} \boldsymbol{U}'(x, y)) \tag{5}$$

where A is the amplitude of the perturbation, $\Re$ denotes the real part and k_{zm} is the most unstable vertical wavenumber obtained by a linear stability analysis [13]. In most simulations, the vertical size of the box L_z is set to the most unstable wavelength $L_z = \lambda_{max} = 2\pi / k_{zm}$.

In order to obtain the basic flow $\boldsymbol{U}_{2D}$, a 2D non-linear simulation is first carried out with the following initial vorticity field corresponding to two identical co-rotating gaussian vortices of initial radius a_i, circulation Γ_i, separated by an initial distance b_i:

$$\omega_i = \frac{\Gamma_i}{\pi a_i^2} \left(\exp\left(- \frac{(x - \frac{b_i}{2})^2 + y^2}{a_i^2} \right) + \exp\left(- \frac{(x + \frac{b_i}{2})^2 + y^2}{a_i^2} \right) \right). \tag{6}$$

This two-dimensional simulation is conducted for the same set of parameters (n_x, n_y, L_x, L_y, ν) as the 3D simulation. Each vortex is deformed by the strain field created by the companion vortex and becomes slightly elliptical [15–18]. Then, the vortex core a increases slowly by diffusion whereas the distance b remains constant. The velocity field $\boldsymbol{U}_{2D}$ is taken during this quasi-steady phase when the ratio a/b has reached the desired initial value a_0/b_0 for the 3D numerical simulation. The two-dimensional simulation is also continued further in order to have a reference simulation to analyze the 3D simulations. The linear stability analysis of the base flow $\boldsymbol{U}_{2D}$ is also conducted in order to find the most unstable vertical wavenumber k_{zm} and eigenmode $\boldsymbol{U}'$ [13]. The eigenmode $\boldsymbol{U}'$ is normalized so that its total energy per unit vertical length scale is equal to unity.

Space and time are non-dimensionalized respectively by the core size a_0 and by the inverse of the vorticity at the center of each vortex $\tau = \pi a_0^2 / \Gamma_0$, where Γ_0 is the circulation of each individual vortex at time t_0. The same notation is kept for the non-dimensional variables for the sake of simplicity. The Reynolds number is defined as $Re = \frac{\Gamma_0}{\pi \nu}$ and the Froude number is $F_h = \frac{\Gamma_0}{\pi a_0^2 N}$. The Schmidt number $Sc = \nu / D$ is set to unity.

3. Qualitative behavior of the pairing of vortices in a stratified flow

The dynamics of the merging of two co-rotating vortices in a linearly stratified flow has been first computed without the sophisticated initial condition described above. The evolution of two gaussian vertical vortices perturbed by a low amplitude 3D white noise has been computed in a cubic box and with a moderate resolution 128^3. The initial ratio between the core size a_i and separation distance b_i is $a_i/b_i = 0.15$. The initial Froude number is $F_h = 1.33$ and the Reynolds number is $Re = 2120$. The size of the domain is $L_x = L_y = L_z = 10\pi a_i$. Fig. 1 shows the temporal evolution of the vertical vorticity. At the beginning of the simulation ($t = 0$), the vortices are columnar and rotate one around the other at angular velocity $\Omega_i = \Gamma_i / (\pi b_i^2)$. At time $t = 478$, the two vortices are displaced symmetrically alternately closer and away along the vertical in a direction making a well defined angle with the line joining the vortex centers. As a result, the distance between the two vortex axes oscillates along the vertical. This perturbation structure is similar to the one associated with the zigzag instability described by Otheguy et al. [13]. At $t = 557$, the pairing of the vortices has occurred in layers where they were brought closer by the instability. These layers alternate with layers where two well-separated vortices are still rotating one around the other. At $t = 955$, merging has eventually occurred at each vertical station. The final vortex displays a variation of core size along the vertical resulting from the desynchronized pairing. This modulated core is surrounded by low intensity spiral arms (yellow contours).

The vertical wavelength that shows up spontaneously is $\lambda/(bF_h) = 0.5$ with a spatial variability of about 16%, in good agreement with the most unstable wavelength of the zigzag instability $\lambda/(bF_h) = 0.64$ predicted by the linear stability analysis by [13]. Furthermore, the instability manifests itself even at finite amplitude, as bending deformations of the vortices in agreement with