European Journal of Mechanics B/Fluids 49 (2015) 438–447

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Global stability and optimal perturbation for a jet in cross-flow

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ARTICLE INFO

Article history:
Available online 12 June 2014

Keywords:
Absolute/convective instability
Bifurcation
Optimal disturbance
Jet in cross-flow
Direct numerical simulation

ABSTRACT

We study the stability of a jet in cross-flow at low values of the jet to cross-flow velocity ratio R using direct numerical simulations (DNS) and global linear stability analysis adopting a time-stepper method. For the simplified setup without a meshed pipe in the simulations we compare results of a fully-spectral code SIMSON with a spectral-element code Nek5000. We find the use of periodic domains, even with the fringe method, unsuitable due to the large sensitivity of the eigenvalues and due to the large spatial growth of the corresponding eigenmodes. However, we observe a similar sensitivity to reflection from the outflow boundary in the inflow/outflow configuration, and therefore we use an extended domain where reflections are minimal. We apply in our studies both modal and non-modal linear analyses investigating transient effects and their asymptotic fate, and we find a transient wavepacket to develop almost identically in both the globally stable and unstable cases. The final results of the global stability analysis for our numerical setup show the critical value of R, at which the first bifurcation occurs, to lie in the range between 1.5 and 1.6.

1. Introduction

The so-called jet in cross-flow (JCF) refers to fluid that exits a nozzle and interacts with the surrounding boundary layer flowing across the nozzle. This case has been extensively studied both experimentally [1–6], theoretically [7,8] and numerically [9–15] over the past decades due to its high practical relevance. Smoke and pollutant plumes, fuel injection and mixing or film cooling are just a few applications. On the other hand, the jet in cross-flow is considered a canonical flow problem featuring complex, fully three-dimensional dynamics that cannot be investigated under simplifying assumptions commonly applied to simpler flows. It makes the JCF a perfect tool for testing numerical methods for studying the stability of fluid flows and simulation capabilities. Recent reviews on this flow configuration are given in Refs. [16,17].

In this work we concentrate on the incompressible flow with the round perpendicular jet of constant diameter and characterise the JCF by three independent non-dimensional parameters: the free-stream and jet Reynolds numbers ($Re_{\delta_0^\star}$, Re_{jet}) and jet to free-stream velocity ratio R, which is the key parameter here. The major flow features are (see Fig. 1): the counter-rotating vortex pair (CVP) in the far field, the horseshoe vortex placed upstream of the jet orifice [18], and vortices shed from the shear layers caused by the interaction of the jet with the cross-flow. There are other features observed at higher values of the cross velocity ratio R, e.g., wake vortices [1] and upright vortices [14]. As the ratio R increases, the flow evolves from a stable (and thus steady) configuration consisting of a (steady) CVP and horseshoe vortex (Fig. 2), through simple periodic shear layer vortex shedding (a limit cycle; Fig. 16) to more complicated quasi-periodic behaviour, before finally becoming turbulent. The breakup of the CVP due to interaction with vortices shed from the shear layer is also illustrated in Fig. 2 in Ref. [15].

Laminar–turbulent flow transition is a classical problem in fluid mechanics. Initially motivated by aerodynamic applications, it is an important phenomenon in many other industrial applications. Originally, hydrodynamic stability was studied by means of linear stability theory investigating the behaviour of infinitesimal disturbances in space and time around some basic flow state. In the so-called local analysis, the exponential growth of linear perturbations is studied at each streamwise position and the distinction between local convective and absolute stability is made [19]. This local treatment is legitimate for parallel and weakly non-parallel flows, but many of the flow configurations developing strong instabilities and eventually exhibiting transition to turbulence (e.g. JCF) are strongly non-parallel. Moreover, they belong to the open flow category, where fluid particles continuously enter and leave the considered domain. Such unstable open flows require global analysis where the evolution of perturbations is considered in the whole physical domain [20]. The

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http://dx.doi.org/10.1016/j.euromechflu.2014.06.001
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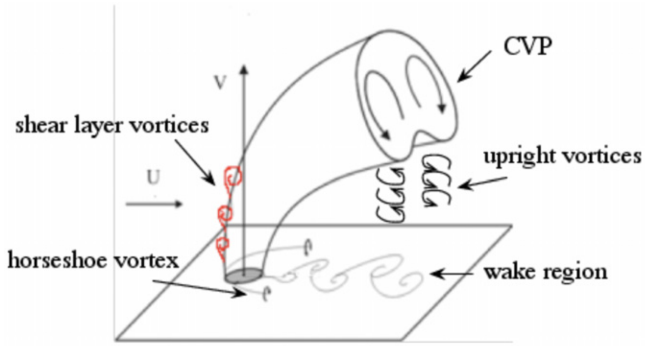

Fig. 1. A sketch of the main vortical structures that may be identified in the jet in cross-flow.
Source: Figure adopted from Ref. [15].

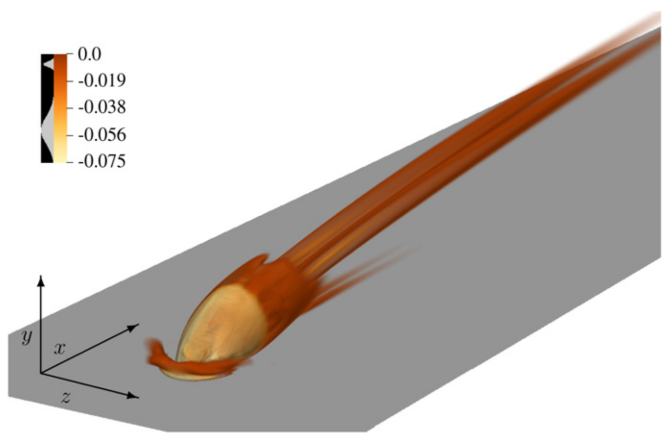

Fig. 2. Vortical structure of the base flow $\vec{U}_b$ for the JCF with $R = 1.5$ obtained with SFD for mesh MS3 and $N = 9$. The steady CVP, shear layer and horse shoe vortex are clearly visible. The vortical structure is presented using volume rendering of the λ_2 vortex identification criterion [39]. Highly negative values of λ_2 are coloured in yellow (vortex 'cores'), and the regions of lower magnitude, i.e., negative value closer to zero, are coloured in brown (vortex 'edges'). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

global behaviour of the flow depends on the competition between local instability and basic advection. The linear global modes are the eigenmodes of the linearised Navier–Stokes operator. The recent review of the work on global stability in the past years can be found, e.g., in Theofilis [21].

The first linear global stability analysis of the JCF in a simplified setup not including the pipe in the computational domain at $R = 3$ was presented by Bagheri et al. [13,14]. In this work the pipe orifice was represented by the Dirichlet boundary conditions. For this jet to free-stream velocity ratio the JCF was found to be dominated by an interplay of three common instability mechanisms: a Kelvin–Helmholtz shear layer instability, a possible elliptic instability of the CVP, and a near-wall vortex shedding mechanism similar to a von Kármán vortex street. It was also shown that the flow acts as an oscillator, with high-frequency unstable global eigenmodes associated with shear-layer instabilities on the CVP and low-frequency modes resulting in vortex shedding in the jet wake. This work was later extended to the wider range of $R \in (0.55, 2.75)$ by Ilak et al. [15], focusing on transition from steady to unsteady flow as R is increased. The first bifurcation i.e., the appearance of the first unstable eigenmode, was found to occur at $R \approx 0.675$, when shedding of hairpin vortices characteristic of a shear layer instability was observed, and the source of this instability (*wavemaker*) was located in the shear layer just downstream of the orifice. Results of linear stability analysis were consistent with nonlinear direct numerical simulations (DNS) at the critical value of R predicting well the frequency and initial growth rate of the disturbance. It was also concluded that, based on linear analysis, good qualitative predictions about the flow dynamics can be made even for higher values of R, where multiple unstable eigenmodes are present. The authors pointed out, however, that the critical value of R cannot be determined exactly due to sensitivity of the results to changes in the domain length as well as to the presence of the fringe region enforcing periodic boundary condition (BC).

In the current study we follow Ilak et al. [15] focusing on the transition from steady to unsteady flow and, using linear global stability analysis, searching for the value of R at which the first bifurcation occurs. The scope of this work is to study global stability of the JCF focusing on the eigenmode sensitivity to the simulation parameters. This way we test the numerical methods and identify the major practical difficulties related to linear stability of this type of complex flows.

However, as purely modal analysis is known to fail in predicting the practically observed critical Reynolds number for transition to turbulence in a number of systems [22–29], we apply in our studies both modal and non-modal analyses. A classical example of such a flow is the convectively unstable flat-plate boundary layer [30], which behaves as broadband amplifier for incoming disturbances, but is globally stable according to linear global analysis. However, a global stability analysis based on the asymptotic behaviour of single eigenmodes of the system does not capture all relevant dynamics, and transition to turbulence at finite *Re* occurs due to transient effects. Following Ref. [31] we investigate the linear growth of perturbations in the JCF for a limited time, before the exponential modal behaviour is most dominant, and determine an *optimal initial condition* (initial condition yielding largest possible growth in energy) adopting a time-stepper method.

This paper is organised as follows. Section 2 describes the numerical methods used for modal and non-modal stability calculations including a brief description of the employed codes. In this section we also give details of computational setup. Section 3 is devoted to the global stability of the JCF. We discuss here results obtained by DNS and linear modal analysis focusing on the sensitivity of bifurcation point to various simulation parameters and employed code. Results of non-modal analysis are presented in Section 4 and the final discussion with conclusions is given in Section 5.

2. Simulation setup and numerical method

We adopt the same computational setup as Ilak et al. [15], modelling the interaction of a boundary layer with a perpendicular jet exiting a circular pipe with diameter $D = 3\delta_0^\star$, where $\delta_0^\star$ is the displacement thickness at the inflow placed $9.375 \cdot \delta_0^\star$ upstream of the centre of the pipe orifice. In our calculations $\delta_o^\star$ is used as reference length unit. Following Ref. [15] we use both a laminar cross-flow and jet inflow profile and, as the jet pipe is absent in our simulations, an inhomogeneous (Dirichlet) BC prescribing the inflow jet profile is employed instead. This is an important limitation of the problem setup requiring, e.g., smoothing of the jet profile by a super-exponential Gaussian function;

$$v(r) = V(1 - r^2)\exp(-(r/0.7)^4),$$

where v is the wall-normal velocity, V is the *peak* jet velocity, and r is the distance from the centre of the jet nozzle (x_{jet}, z_{jet}), defined as:

$$r = (2/D)\sqrt{(x - x_{jet})^2 + (z - z_{jet})^2}.$$

A full discussion on this choice of profile can be found in Refs. [13,15].