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Laminar boundary-layer separation control by Görtler-scale blowing

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ABSTRACT

Recent experimental work has succeeded in retarding or removing boundary-layer separation by means of blowing supersonic microjets transversely through the wall. To provide some theoretical context for such work, the current study examines the removal of separation by transverse blowing within the framework of the standard Prandtl scalings for incompressible boundary layers. One key result, obtained using asymptotic analysis, is that such removal is not possible for two-dimensional flow. Neither is removal of separation possible by three-dimensional blowing in an initially two-dimensional separated boundary layer if the blowing distribution has a finite-scale spanwise variation. The second key result obtained is that the previous conclusion is no longer valid when there is nontrivial short-scale spanwise variation of the blowing distribution. This result is obtained by providing a numerical counter-example in which blowing, with a Görtler scale spanwise variation, creates an attached boundary layer flow where none existed before the blowing. One consequence is that there are at least some flows in which transverse Görtler-scale blowing can turn a separated flow into an attached flow, with a vanishingly small drag that is inversely proportional to the square root of the Reynolds number. The flow physics of the computed example is analyzed to obtain a better understanding of how the Görtler-scale blowing affects the flow.

1. Introduction

Effective control of boundary-layer separation can have many benefits. For example, avoidance of stall limits helicopter rotor efficiency and performance, especially on the retreating blades in the presence of forward motion. Stall is also a limiting factor for control surfaces of missiles and projectiles. A large number of control mechanisms have been developed, ranging from classical suction and vortex generators to synthetic jets, and some are quite effective. The largest problem is often not efficiency but associated costs and practical application in challenging real-life environments.

A new approach has been proposed recently that promises to be much more robust and effective in applications. Control is achieved by blowing supersonic microjets, with diameters described in microns, into the boundary layer. Experiments for both dynamic stall, [1], and steady separation, [2–6], show that the microjets are effective in eliminating stall and its adverse effects on lifting forces and resistance. These are very promising results, because unlike classical suction, blowing will repel dirt and other contaminants, rather than suck them toward the surface. See [7,8] for more on such issues. Moreover, sources of high pressure air, such as engine bleed, may readily be found. Because of the micron dimensions, the amount of air required by the microjets is negligible. By its nature, the control can readily be completely removed when no longer needed and it is easily modulated, [9,10].

However, optimizing the location, spacing and distribution of the jets to predict and maximize benefits without prohibitive situation-specific experiments is a significant problem due to a lack of understanding of why the control is effective. While the generation of enhanced streamwise vorticity seems to be a likely mechanism for the beneficial effects, the process is clearly internal to the boundary layer, due to the microscopic size and mass flow rates of the jets, [2–6]. Modeling the process as classical vortex generators that produce organized vortices of significant scale is simply not realistic.

For those reasons, it seems worthwhile to look for a simple model that may explain some of the issues involved in microjet separation control. The simplest reasonable model would seem to be two-dimensional laminar boundary layer flow with distributed boundary-layer scale blowing through the wall. Of course, this model will not describe the precise small-scale features of the flow right at the microjets. However, the actual separation being removed is well downstream of the microjets, where the small scale details of the jets are presumably long diffused out. So the model seems a reasonable starting point. And there is appreciable existing data on the effects of blowing and suction within this model.

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But the first problem now arises immediately. Since the pioneering paper of Prandtl [11] that initiated boundary layer theory, the notion has been established that transverse suction, rather than blowing is needed to remove separation. Indeed, considerable practical experience in two-dimensional laminar boundary layer computations, (e.g. [12,13]), suggests that transverse blowing promotes, rather than prevents, separation.

The question then becomes whether it is even possible, within the two-dimensional model, to remove separation by blowing, regardless of how well the blowing distribution is chosen to simulate microjets. In Section 3 we show that the answer is no. Separation cannot be removed by blowing in a two-dimensional laminar incompressible Prandtl boundary layer.

This result is interesting, because it indicates that microjet flow control is not as trivial as it may seem. (Consistent with that, Vikas Kumar, during his Ph.D. thesis defense noted that it was possible to create separation using microjets where there was none before.) Furthermore, the result generalizes to the statement that using a three-dimensional blowing distribution cannot remove separation either, as long as the distribution has a finite spanwise scale.

However, when the spanwise variation of the blowing distribution becomes sufficiently small, the given analytical arguments that exclude removal of separation are no longer valid. The question becomes then whether it remains impossible to remove separation using transverse blowing on a boundary-layer scale. In Section 6 it is shown by a counter-example that the answer is no. The counter-example removes separation from a slightly concave surface by blowing on a short, Görtler-type, spanwise scale.

The counter-example gives a reasonable qualitative explanation of the experimental results of [2–4] within the simple framework of incompressible laminar boundary layer theory. (Note that the experiments were turbulent.) As discussed in Section 8, considerable further efforts seem to be needed to gain a better understanding of other cases in which microjets have been used.

2. Comments on the definition of separation

One issue that seems to require clarification is what we mean with the terms "separated" and "unseparated" flow. Prandtl's classical criterion that separation starts at zero wall shear $\tau_x = 0$ was derived for two-dimensional flow, [11]. The present paper, however, deals with three-dimensional flows, and in addition the spanwise scales are small rather than finite in our flows. Some authors have suggested using $n_x\tau_x + n_z\tau_z$, with x, z the wall plane and n_x, n_z the unit vector normal to the separation line as the criterion for separation in three-dimensional flow. (This would presumably become $\tau_x = 0$ at the first point of separation.) One other suggestion we received is that we should instead expect a separation due to the spanwise flow of the type whose asymptotic behavior was described by Stewartson and Simpson [14]. (Actually, this separation *does* have zero wall shear in boundary layer approximation. However, for the similar Banks and Zaturska [15] type of separation process, which might be expected to occur in steady flow for say a wall jet inside a curved pipe, the streamwise wall shear could be anything. That was shown numerically by Van Dommelen [16].)

However, in Appendix B, we provide numerical results that suggest quite strongly that separation does *not* occur at the first wall point with streamwise wall shear zero. We do not use zero wall shear, in any direction, as a criterion for separation. Instead, we have long adhered to the view first explicitly expressed by Sears and Telionis [17]. Since this view is not that well known, we will give a review here.

Already in his pioneering study in 1904, Prandtl [11] had identified zero wall shear as the criterion of steady separation from a fixed wall. This criterion subsequently became widely established as a convenient definition of separation in general. However, in the 1950s, a number of authors, including Moore [18], Rott [19], and Sears [20], (MRS), had expressed concerns about the physical meaning of the criterion in unsteady flows, and in steady flows over moving walls.

Generalizing the earlier work by Moore [18], Sears' Ph.D. student Telionis revisited the question in the 1970s. Based on a study that some called more philosophical than mathematical, Sears and Telionis [17] proposed a generalization of Prandtl's criterion: the separation point would still be at zero wall shear, but not necessarily at the wall. They proposed that in general, the separation point would move with the local flow velocity. This reduces to Prandtl's original condition for steady separation from a fixed wall: in steady flow a separation point cannot move and it is the fluid at the wall that is at rest. Sears & Telionis dubbed the generalized conditions the MRS conditions.

The theory received some support when various early boundary layer computations of relevant flows showed the MRS conditions to apply, [21]. However, these early solutions were subject to the criticism that the prescribed external flow was inconsistent with a separated flow. And there was more criticism. For one, some argued that Prandtl's criterion of zero wall shear remained "convenient" even if the separation was unsteady. More significantly, it was noted that the MRS conditions are incomplete. To apply the MRS conditions to find the separation point requires knowledge of the velocity of the separation point. Now in steady flows that velocity is zero, and in flows with symmetries, like semi-similar flow, it can be deduced from the symmetry. But in general unsteady flows, it requires a priori knowledge of the position of the separation point versus time, the very thing that was to be found.

But Sears & Telionis had an answer to all these criticisms. In 1948, Goldstein [22] had addressed issues in previous numerical work, that computed boundary layers at Prandtl's point of zero wall shear. He showed that at such a point a self-consistent *singular* solution exists, in good agreement with earlier computations by Hartree. Noting this Goldstein singularity, Moore [18] wrote "Of course, the full Navier–Stokes equations do not show such a singularity. However, the existence of *a* singular boundary-layer solution is no doubt a reliable indication of separation, insofar as the boundary-layer equations are able to describe it." (Emphasis added.) Sears and Telionis [17] inverted that: "[…] that the appearance of the Goldstein singularity, *modified as necessary*, in the solution of the boundary layer equations, be adopted as the most general *definition* of separation". (Emphasis added.)

This proposal came under much greater criticism still than the MRS conditions, and from two groups. The first group argued that the Navier–Stokes equations do not have a singularity, and that the interest was in the solution of the Navier–Stokes equations, not the boundary-layer equations. However, this criticism does not allow for much meaningful mathematical analysis in fluid mechanics. As a simple example, consider the case of the thin Blasius' boundary layer along a flush flat plate at large Reynolds number, [23]. Taken literally, that case is nonsensical: mathematics knows no subjective terms like "thin" and "large". Instead it has *limit processes*. In such a setting, "thin" really means that the limit is zero, and "large" that the limit is infinite. Limit processes *require* that the problems are embedded in a larger setting than just a single example flow.

In particular, the limit process relevant for flows like those in this paper is where the Reynolds number is allowed to go to infinity. And that almost unavoidably brings in singularities. The Blasius boundary layer above is rigorously defined as the "jump in flow velocity at the wall at infinite Reynolds number", a singularity. The Blasius velocity profile is rigorously defined as "the limiting flow velocity in suitably rescaled coordinates for infinite Reynolds number", which is nonsingular in this case.

The second group that strongly criticized the singularity proposal consisted of theoreticians. They were familiar with the central role of singularities in any meaningful analysis of fluid flows.