European Journal of Mechanics B/Fluids 46 (2014) 37–45

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Fully developed mixed convection flow in a horizontal channel filled by a nanofluid containing both nanoparticles and gyrotactic microorganisms

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HIGHLIGHTS

- The passively controlled mathematical model for nanofluids is introduced, which could be physically more realistic than the previous models.
- The influence of nanoparticles on the bioconvection is investigated.
- An improved HAM technique for nonlinear problems with complicated boundary conditions is developed.

ARTICLE INFO

Article history:
Received 3 December 2013
Accepted 10 February 2014
Available online 15 February 2014

Keywords:
Nanofluid
Horizontal channel
Bioconvection
Linear boundary conditions

ABSTRACT

In this paper, an analysis is made for the fully developed mixed bioconvection flow in a horizontal channel filled with a nanofluid that contains both nanoparticles and gyrotactic microorganisms. The passively controlled nanofluid model proposed by Kuznetsov and Nield (2013) is then introduced for modeling this flow problem, which is found to be more physically realistic than previous nanofluid models. Analytical approximations with high precision are obtained by the improved homotopy analysis technique for complicated boundary conditions. Besides, the influences of various physical parameters on the distributions of temperature, the nanoparticle volume fraction, as well as the density of motile microorganisms are investigated in detail.

1. Introduction

The interest of flow and heat transfer in the field of nanofluids has been stimulated significantly in recent years due to its numerous potential applications in industrial processes such as in power generation, chemical processes and heating or cooling processes. Choi [1] was the first who used the term 'nanofluids' to describe the pure fluids with suspended nanoparticles. He found that the thermal conductivity of the traditional heat transfer fluids can be enhanced dramatically as appropriate metallic particles in nanosize are suspended into the base fluids. Buongiorno [2] noticed that when turbulent effects are absent, the Brownian diffusion and the thermophoresis are the most important factors for determination of the behaviors of nanofluids. He then established the conservation equations based on these two effects. Also based on these two dominant factors, Nield and Kuznetsov [3] and Kuznetsov and Nield [4] introduced Buongiorno's [2] model into the well-known Cheng–Minkowycz problem [5] and the problem of the natural convective flow past a vertical plate. These two works gained applausive successes in the community of heat transfer and their idea was further used by many researchers for various heat transfer problems such as [6–9]. Several excellent books [10,11] and review papers [12–16] on convective heat transfer in nanofluids have been carried out for description of the characteristics of nanofluids in the past few years.

Many aspects of bioconvection problems in suspensions that contain solid particles were investigated by several researchers such as Kuznetsov and Avramenko [17], Geng and Kuznetsov [18–20], Kuznetsov [21], Kuznetsov and Geng [22], and Kuznetsov [23,24]. Very recently, Kuznetsov [25–27] made a series of analysis on the bioconvection of a nanofluid in a suspension containing both nanoparticles and microorganisms. He noticed that the addition of gyrotactic microorganisms into nanofluids is likely to increase

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http://dx.doi.org/10.1016/j.euromechflu.2014.02.005
0997-7546/

Nomenclature

a	positive stretching constant
b	chemotaxis constant
C	nanoparticle volume fraction
C_0	nanofluid volume fraction of the lower wall
D_B	Brownian diffusion coefficient
D_n	diffusivity of microorganisms
D_T	thermophoretic diffusion coefficient
$f(\eta)$	dimensionless stream function
$\mathbf{j}$	flux of microorganisms
L	distance
Le	Lewis number
N	number density of motile microorganisms
N_1	density of microorganisms at the lower wall
N_2	density of microorganisms at the upper wall
N_b	Brownian motion parameter
N_t	thermophoresis parameter
p	pressure
Pe	Péclet number
Pe_b	bioconvection Péclet number
Pr	Prandtl number
Re	Reynolds number
$s(\eta)$	similarity function for the number density of motile microorganisms
Sc	Schmidt number
T	temperature inside the boundary layer
T_1	temperature of the lower wall
T_2	temperature of the upper wall
u, v	velocity components along the x- and y-axes, respectively
$\hat{\mathbf{v}}$	average swimming velocity vector of the oxytactic microorganisms
$\mathbf{v}$	velocity vector
W_c	maximum cell swimming speed
x, y	Cartesian coordinates along the surface and normal to it, respectively

Greek symbols

α	thermal diffusivity of the nanofluid
$\delta_s, \delta_\phi, \delta_\theta$	constants
$\phi(\eta)$	dimensionless nanoparticle volume fraction
η	similarity variable
$\theta(\eta)$	dimensionless temperature
ν	kinematic viscosity
ρ_f	nanofluid density
$\rho_f c$	heat capacity of the fluid
$\rho_f c$	effective heat capacity of the nanoparticle material
τ	ratio of the effective heat capacity of the nanoparticle to that of the fluid
ψ	stream function
ζ	vorticity function

its stability as a suspension. He further found that suspensions of gyrotactic microorganisms could exhibit bioconvection, which is a macroscopic motion in the fluid induced by up swimming or the motion of motile microorganisms. This is due to that the motile microorganisms are usually heavier than water so that they can swim in the upward direction in response to stimuli such as gravity, light and chemical attractions. Kuznetsov [28] also found that the movement of motile microorganisms can cause an unstable top heavy density stratification which leads to development of hydrodynamic instability. Kuznetsov [29] observed that the thermo-bioconvection due to gyrotactic microorganisms could be important for understanding fluid motion in hot springs populated by motile thermophiles (heat loving microorganisms). Such thermo-bioconvection has great potentials in applications of modeling oil and gas-bearing sedimentary basins [30] and microbial enhanced oil recovery [31,32]. It is worth mentioning to this end that according to Kuznetsov [33], bioconvection may have contributed toward the biomicrosystems that are for the mass transport enhancement and mixing, which are important issues in many microsystems. On the other hand, a nanofluid in general is accepted as a good thermal conductivity medium. Therefore, combining both advantages of bioconvection and thermal conductivity of nanofluid, the heat transfer rate is expected to improve significantly.

The aim of the present study is to investigate the fully developed mixed convection flow between two paralleled horizontal flat plates filled by a nanofluid containing both nanoparticles and gyrotactic microorganisms. The present study is to extend the work done by Kuznetsov [24] to the case of fully developed flow in a horizontal channel with stretching walls. The passively controlled nanofluid model proposed by Kuznetsov [34] is applied, which could be physically more realistic than previous nanofluid models. By means of similarity reductions, a set of four coupled nonlinear equations with linear boundary conditions is obtained. This particular kind of nonlinear equations are solved analytically. Besides, the effects of the governing parameters on the temperature, the nanoparticle volume fraction, as well as the density of motile microorganisms profiles are graphically presented and discussed. As far as we know, this problem is never considered before so the results are new and original.

2. Basic equations

Consider a fully developed mixed bioconvection flow of a nanofluid that contains both nanoparticles and gyrotactic microorganisms between two paralleled horizontal flat plates separated by a distance $2L$. We select a coordinate system in which the x-axis is along the centerline of the channel and the y-axis is orthogonal to the channel walls. It is assumed that both the walls are stretched by the same velocity $u = ax$. It is also assumed that the walls have the constant temperature distributions with the temperature on the upper wall being T_2 and on the lower wall being T_1. The nanoparticles fraction on the upper wall is assumed to obey the passively controlled model proposed by Kuznetsov and Nield, while the nanoparticles distribution on the lower wall is assumed to be a constant C_0. Similar to the temperature distribution, here we assume that the microorganisms have the constant distributions with the density of the microorganisms on the upper wall being N_2 and on the lower wall being N_1. It is worth mentioning that the microorganisms can only survive in water. This indicates that the base fluid to be considered has to be water. On the other hand, the nanoparticles suspended in the base fluid are assumed to be stable and do not agglomerate in the fluid. The nanoparticles are also assumed to have no effect on the swimming direction and velocity of the microorganisms. Furthermore, to avoid bioconvection instability due to the increase of the suspension's viscosity, the nanofluid used here has to be dilute. With those assumptions, the following five field equations embodying the conservation of total mass, momentum, thermal energy, nanoparticle volume fraction and microorganisms are given, using the nanofluid model proposed by Kuznetsov and Nield [34], in the following forms:

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\rho_f (\mathbf{v} \cdot \nabla) \cdot \mathbf{v} = -\nabla p + \mu \nabla^2 \mathbf{v}, \tag{2}$$

$$\mathbf{v} \cdot \nabla T = \alpha \nabla^2 T + \tau [D_B \nabla T \cdot \nabla C + (D_T/T_\infty) \nabla T \cdot \nabla T], \tag{3}$$

$$(\mathbf{v} \cdot \nabla) C = D_B \nabla^2 C + (D_T/T_0) \nabla^2 T, \tag{4}$$

$$\nabla \cdot \mathbf{j} = 0, \tag{5}$$