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Effects of tank cross-section on dynamic fluid slosh loads and roll stability of a partly-filled tank truck

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HIGHLIGHTS

- An analytical model of lateral slosh is developed using the linear slosh theory.
- The model is applicable to tanks with arbitrary cross-sections and liquid depths.
- Effectiveness of Reuleaux-triangle tank in suppressing the liquid slosh is shown.
- Rollover limits are much lower than those of the widely-used quasi-static analysis.
- Highest rollover limits are obtained for vehicle with the Reuleaux-triangle tank.

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ABSTRACT

An analytical model of a partly-filled tank of arbitrary cross-section is developed for predicting transient lateral slosh force and overturning moment using linear slosh theory. Slosh frequencies and mode shapes are initially estimated using the variational method, which is applied to the linearized free-surface boundary condition. The resulting truncated system of linear ordinary differential equations is subsequently solved numerically to determine the fluid velocity potentials followed by hydrodynamic force and moment. The validity of the model is examined through comparisons with available analytical solutions and experimental data. The slosh force and roll moment are obtained for four different tank cross-sections, namely, circular, elliptical, modified-oval and Reuleaux-triangle. It is shown that the magnitudes of the slosh force and overturning moment are strongly dependent upon the tank cross-section. The slosh model is subsequently integrated to a roll plane model of an articulated tank-semitrailer vehicle to study the effect of dynamic liquid slosh as well as the tank cross-section on the steady-turning roll stability limit of the vehicle under constant and variable cargo load conditions. The results suggest that a tank cross-section with lower overall center of mass and lower critical slosh length yields an enhanced roll stability limit under medium- and high-fill conditions.

1. Introduction

The directional responses of heavy vehicles carrying liquid cargo are known to be strongly affected by liquid slosh caused by road- or maneuver-induced disturbances. The forces and moments caused by fluid slosh under steering and/or braking maneuvers have been associated with reduced stability limits and poor directional performance of partly filled tank trucks [1,2]. Directional responses and roll stability of partially filled tank trucks have been mostly evaluated using quasi-static fluid slosh theory, which neglects the contributions due to dynamic sloshing. The steady-turning rollover immunity levels of articulated vehicles with partly filled cleanbore and compartmented cylindrical tanks have been evaluated using a static roll plane model of the vehicle integrating forces and moments arising from the quasi-static fluid slosh [3]. The study concluded that the partially filled cleanbore tank vehicles exhibit lower rollover threshold acceleration compared with the rigid cargo vehicles. The quasi-static fluid slosh models have also been employed to obtain directional responses of partly filled tractor–tank-semitrailer combinations under steering or braking inputs [4,5]. Southcombe et al. [6] developed a simple numerical model for estimating the steady-turning roll stability limits

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of partly filled tank vehicles with various tank geometries using quasi-static fluid slosh analysis.

The quasi-static method permits analyses of mean steady-state directional responses of the coupled cargo–vehicle system in a highly efficient manner through computations of mean dynamic load shifts, mean slosh forces, and moments. The method, however, cannot account for the effects of dynamic liquid slosh. Studies reporting dynamic fluid slosh in partly filled containers using computational fluid dynamics (CFD) codes have invariably concluded that peak slosh forces and moments are substantially greater than those predicted from a quasi-static model [2,7,8]. These suggest only limited validity of the quasi-static method for predicting directional responses of the vehicles, particularly under transient directional maneuvers or moderate-to-high amplitude steering or braking inputs. It has thus been widely suggested that considerations of transient fluid slosh could yield lower stability limits of partly filled tank trucks compared with those estimated from the quasi-static analyses. Moreover, the quasi-static approach cannot be applied to study fluid slosh in baffled tanks.

The applications of CFD slosh models to vehicle models for braking and steering dynamic analysis, however, have been limited due to extreme computational demands and complexities involving elaborate data transfers and coordinate transformations between the vehicle and the CFD fluid slosh models. A few studies have attempted analyses of coupled tank–vehicle system dynamics through continuous updating of the vehicle state on the basis of transient fluid slosh forces and moments derived from a CFD fluid slosh model, in an off-line manner [1,9,10]. Thomassy et al. [11] developed a master program, called Glue Code, for managing data transfer between the CFD (FLOW-3D) and multi-body dynamic (MSC.ADAMS) codes, which simulated the liquid slosh and vehicle dynamics, respectively, in an independent manner. Such methods, however, are considered highly inefficient due to excessive computing demands [9–11].

Alternatively, the dynamic fluid slosh in moving containers under lateral and longitudinal accelerations may be characterized through mechanical analogous models. Such models can be easily integrated to the vehicle model for directional response analyses in a highly efficient manner. In the mechanical analogous modeling approach, the dynamic motion of liquid within a container is represented by a series of mass–spring–dashpot systems or a set of simple pendulums [12,13]. While the simplicity of the mechanical equivalent models could permit even real-time simulations of directional responses of a partly filled tank vehicle, the identification of equivalent mechanical model parameters poses certain complexities. The model parameters are generally estimated from experimental data [14] or slosh responses obtained through CFD simulations [15].

Analytical models of liquid slosh, based on the potential flow equations and linearized free-surface boundary condition, are known to be efficient in terms of their computational demand and ease of integration to vehicle dynamic models. Ibrahim [16] suggested that exact solutions of linear liquid slosh are limited to tank geometries with straight walls, such as rectangular and upright-cylindrical containers. Analytical solutions of fluid slosh in tanks with curved walls have also been attempted in a few studies using a conformal mapping technique. Budiansky [17] and McIver [18] used this method to calculate natural sloshing modes and frequencies of liquid sloshing in two-dimensional cylindrical and spherical containers as a function of liquid fill depth. Hasheminejad and Aghabeigi [19,20] computed natural frequencies and modes of two-dimensional slosh in a half-filled elliptical container with vertical or horizontal side baffles. For lateral liquid slosh in containers with curved walls, analytical solutions have been mostly limited to half-full condition. Papaspyrou et al. [21] investigated liquid sloshing in half-full cylindrical vessels subjected to transverse excitations considering the velocity potential as a series of spatial and time functions. Hasheminejad and Aghabeigi [20,22] employed the conformal mapping technique to obtain slosh force and overturning moment in half-filled elliptical containers equipped with vertical or horizontal side baffles under a lateral acceleration excitation. The ranges of applicability and limitations of the linear slosh theory have been presented in a recent study [23]. It was shown that the linear slosh theory can provide accurate predictions of transient lateral slosh, when the free-surface elevation is less than the critical free-surface elevation. Comparisons with the nonlinear CFD simulations also suggested reasonably good predictions of slosh force and moment responses under lateral acceleration magnitude of up to $0.4g$, which represents the upper limit of the rollover threshold acceleration for the vast majority of tank truck configurations.

The primary motivation of this study arises from the desire to develop an efficient tool for real-time simulations of fluid slosh in partly filled tanks of different cross sections and to investigate the effect of dynamic fluid slosh on the directional performance of partly filled tank vehicles. An analytical model is proposed to characterize dynamic fluid slosh in moving containers of different cross sections using modal analysis technique. The eigenvalues and eigenvectors of free liquid slosh in a partially filled tank of arbitrary cross section are initially obtained using the variational method. The estimated eigenvalues and eigenvectors are subsequently incorporated into a multimodal representation of lateral liquid slosh to compute the velocity potential followed by the hydrodynamic pressure, slosh force, and moment. The results are presented for the partly filled circular, elliptical, modified-oval, and Reuleaux-triangle [24] cross-section tanks. The validity of the model is also illustrated using the reported measured data and analytical benchmarks. The effect of tank cross section on the lateral slosh force and overturning moment is also investigated. Moreover, a simple roll plane model of a tank vehicle is used to illustrate the effect of dynamic liquid slosh and tank cross section on the rollover threshold acceleration of the partly filled tank vehicle.

2. Formulations

An analytical model of fluid slosh within a tank of arbitrary cross section is formulated to facilitate its integration to the multi-body vehicle dynamic models. The model is developed using the linear modal theory, which can describe velocity potential of the sloshing liquid through various slosh modes. The proposed multimodal approach is supported by the fact that natural modes of liquid slosh satisfy the Laplace equation as well as the zero Neumann boundary condition on the tank walls. The analytical model of lateral liquid slosh in an arbitrary tank cross section involves determination of slosh frequencies and modes, and their summation to derive the velocity potential and thus the slosh force and moment.

2.1. Natural sloshing modes and frequencies

Consider a partly filled arbitrary cross-section tank of overall width $2a$ and height $2b$ fitted within an open rectangular tank, as shown in Fig. 1. The Cartesian coordinate system Oyz, located at the center of the tank, is used with the z-axis directed upward, and the liquid fill level h is measured from the y-axis. As the eigenfunctions of free liquid slosh can be derived analytically for the rectangular tank, these functions are used for a problem with more complex geometry. Assuming inviscid, incompressible, and irrotational flows, the problem of free linear slosh in partially filled tanks can be expressed in the form [16,25]

$$\nabla^2 \varphi_i = 0 \quad \text{in } \Omega \tag{1}$$

$$\frac{\partial \varphi_i}{\partial n} = 0 \quad \text{on } \tau \tag{2}$$

$$\frac{\partial \varphi_i}{\partial z} = \kappa_i \varphi_i \quad \text{on } \Sigma. \tag{3}$$