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Statistics of vorticity alignment with local strain rates in turbulent premixed flames

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HIGHLIGHTS

- The statistics of alignment of vorticity with local principal strain rates have been analysed.
- Effects of regime of combustion and the global Lewis number have been investigated.
- Relative alignments with local principal strain rates are affected by Damköhler and Lewis numbers.
- Detailed physical explanations have been provided for the aforementioned observed behaviours.

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ABSTRACT

The instantaneous alignment of the vorticity vector with local principal strain rates is analysed for statistically planar turbulent premixed flames with different values of heat release parameter and global Lewis number spanning different regimes of combustion. It has been shown that the vorticity vector predominantly aligns with the intermediate principal strain rate in turbulent premixed flames, irrespective of the regime of combustion, heat release parameter and Lewis number. However, the relative alignment of vorticity with the most extensive and compressive principal strain rates changes based on the underlying combustion conditions. Detailed physical explanations are provided for the observed behaviours of vorticity alignment with local principal strain rates. It has been shown that heat release due to combustion significantly affects the alignment of vorticity with local principal strain rates. However, the mean contribution of the vortex-stretching term in the transport equation of enstrophy remains positive for all cases considered here, irrespective of the nature of the vorticity alignment.

1. Introduction

The alignment of the vorticity vector with local principal strain rates is of fundamental importance for the understanding and modelling of turbulent fluid motion, as the alignment statistics directly affect the nature of the vortex-stretching mechanism [1]. It has been demonstrated in several previous studies that the vorticity vector instantaneously aligns with the intermediate eigenvector of strain rate tensor for non-reacting turbulence [2–12]. However, relatively limited attention was given to the analysis of alignment of vorticity with local strain rates in the case of turbulent reacting flows [13–15]. In many applications (e.g. Spark Ignition (SI) engines and industrial gas turbines), the fuel and oxidiser are homogeneously mixed prior to the combustion process (i.e. premixed combustion). Thus, the understanding of vorticity alignment with local principal strain rates is of fundamental interest for the development of high-fidelity models which can, in turn, contribute to the design of new generation energy-efficient and environment-friendly combustion devices. The analysis of Nomura and Elghobashi [13], Boratov et al. [14] and Jaberi et al. [15] concentrated on vorticity alignment with local principal strain rates for non-premixed flames where fuel and oxidiser are completely separated from each other prior to the combustion process. Recently, Hamlington et al. [16] analysed vorticity statistics in premixed combustion based on numerical solutions of reactive systems. The analysis by Nomura and Elghobashi [13] demonstrated that the vorticity vector aligns with the intermediate principal strain rate in non-premixed flames similar to non-reacting turbulent flows but vorticity in non-premixed flames shows appreciable probabilities of local alignment with the most extensive principal strain rate. The analysis by Boratov et al. [14] on non-premixed flame DNS data reveals that the extent of vorticity alignment with the most extensive principal strain rate increases in the regions where the magnitude of strain rate dominates over the vorticity magnitude. By contrast, vorticity shows preferential alignment with the intermediate principal strain rate in the regions where the vorticity magnitude dominates over the strain rate magnitude. The analysis by Jaberi et al. [15] further demonstrated that the alignment of

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Nomenclature

Arabic

a	Acoustic velocity
a_{chem}	Strain rate induced by chemical reaction
a_{turb}	Turbulent straining
a_T	Tangential strain rate
A_i	ith wave associated with boundary
B	Pre-exponential factor
B^*	Normalised pre-exponential factor
c	Reaction progress variable
c^*	Reaction progress variable value indicating the flame surface
C_P	Specific heat capacity at constant pressure
C_V	Specific heat capacity at constant volume
D	Progress variable diffusivity
Da	Damköhler number
e_α	Most extensive principal strain rate
e_β	Intermediate principal strain rate
e_γ	Most compressive principal strain rate
e_θ	Principal strain rate
e_{ij}	Component of strain rate tensor
$\hat{e}_1, \hat{e}_2, \hat{e}_3$	Eigenvectors associated with eigenvalues e_α, e_β and e_γ respectively
E_{ac}	Activation energy
f_1	Terms involving viscosity gradients in the vorticity transport equation
f_2	Terms involving viscosity gradients in the enstrophy transport equation
k	Thermal conductivity
Ka	Karlovitz number
l	Integral length scale
L_i	ith wave amplitude variation
Le	Lewis number
N_i	ith component of flame normal vector
p	Pressure
p^{req}	Target value of pressure at the boundary
Pr	Prandtl number
q_{Ti}	Conduction heat flux in the ith direction
q_{Ci}	Diffusive mass flux in the ith direction
Q	General quantity
Re_t	Turbulent Reynolds number
S_d	Displacement speed
S_L	Unstrained laminar burning velocity
t	Time
t_{chem}	Chemical time scale
t_f	Initial turbulent eddy turnover time
t_{sim}	Simulation time
t_η	Kolmogorov time scale
T	Non-dimensional temperature
T_0	Unburned gas temperature
T_{ad}	Adiabatic flame temperature
T^{req}	Target value of non-dimensional temperature at the boundary
$\hat{T}$	Dimensional temperature
u_i	ith component of fluid velocity
u_i^{req}	Target value of ith component of fluid velocity at the boundary
u'	Root mean square turbulent velocity fluctuation magnitude
$\vec{u}$	Velocity vector
v_η	Kolmogorov velocity scale
V	Volume
$\dot{w}$	Chemical reaction rate
x_i	ith Cartesian co-ordinate
Y_R	Reactant mass fraction
Y_{R0}	Reactant mass fraction in unburned gases
$Y_{R\propto}$	Reactant mass fraction in fully burned gases

Greek

α_H	Heat release parameter
α	Angle between vorticity and the most extensive principal strain rate
α_p	Angle between pressure gradient and the most extensive principal strain rate
β	Angle between vorticity and the intermediate principal strain rate
β_p	Angle between pressure gradient and the intermediate principal strain rate
β_Z	Zel'dovich number
γ	Angle between vorticity and the most compressive principal strain rate
γ_p	Angle between pressure gradient and the most compressive principal strain rate
γ_G	Ratio of specific heat capacities
δ_{th}	Thermal flame thickness
$\varDelta$	DNS grid spacing
η	Kolmogorov length scale
λ	Thermal conductivity
λ_i	Wave velocity associated with ith wave amplitude variation L_i
$\varLambda$	Vortex-stretching term
μ	Dynamic viscosity
μ_0	Dynamic viscosity of the unburned gas
θ	Angle
φ_1	Function of Lewis number related to a_{chem}
φ_2	Function of Lewis number related to $\partial u_i/\partial x_i$
ρ	Gas density
ρ_0	Unburned gas density
σ_i	Relaxation factor associated with ith wave amplitude variation
τ	Heat release parameter
τ_{ij}	Components of viscous stress
ω_i	ith component of vorticity
$\vec{\omega}$	Vorticity vector
$\varOmega$	Enstrophy (i.e. $\varOmega = \omega_i\omega_i/2$)

Symbol

$\langle Q \rangle$	Ensemble averaged values of a general quantity Q conditionally averaged in bins of c values
$\vec{A} \bullet \vec{B}$	Scalar product between vectors $\vec{A}$ and $\vec{B}$

Acronyms

DNS	Direct Numerical Simulation
LES	Large Eddy Simulation
pdf	Probability density function
RANS	Reynolds Averaged Navier–Stokes

vorticity with the intermediate (most extensive) principal strain rate decreases (increases) due to chemical heat release in non-premixed flames. It is worth nothing that the analysis by Jaberi et al. [15] was carried out for constant volume homogeneous turbulence but their findings were found to be qualitatively similar to the results by Nomura and Elghobashi [13] for non-premixed combustion in the presence of inhomogeneous turbulence. Moreover, Jaberi et al. [15] showed that vorticity remains mostly perpendic-