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Water wave diffraction by a bottom-mounted circular cylinder clamped to an elastic plate floating on a two-layer fluid

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ABSTRACT

The wave diffraction by a bottom-mounted circular cylinder, which is clamped to the center of a floating circular thin elastic plate, in the two-layer fluid of finite depth is investigated for the time-harmonic incident waves of the surface and interfacial wave modes. Each fluid layer is inviscid, incompressible and of constant density. The flexural–gravity waves are composed of the propagating, decaying propagating and evanescent wave modes. Within the framework of the linear potential flow theory, a closed system of simultaneous linear equations is derived to solve the undetermined expansion coefficients with the methods of the angular eigenfunction expansion and the inner product. Explicit numerical computations are employed to test the convergence of the two series for the angular expansions and the evanescent wave modes. The horizontal forces and the moments exerted on the circular cylinder due to different wave modes are discussed in the case of the incident waves of either the surface or interfacial wave mode. It is obtained that the evanescent wave modes are appreciable parts for a high frequency.

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1. Introduction

In recent decades, a variety of offshore structures designed for exploiting ocean resources have attracted close attention among scholars and engineers in several coastal countries. The forces and moments acting on the structures due to wave loads may be appreciable. Owing to the difference of temperature, salinity and density, the ocean may emerge a stratification phenomenon. The middle pycnocline is often idealized as a sharp and stable interface, above and below which the fluids are assumed to be of different constant densities.

For the model of the two-layer fluid, there are two real wave numbers, derived from the dispersion relation, corresponding to the propagating waves of the surface and interfacial wave modes. When the waves encounter an obstacle, the energy may transfer between the two propagating wave modes. Linton and McIver [1] employed the multipole expansions to study the two-dimensional problems concerning the wave scattering by a horizontal circular cylinder in either the upper or lower layer. It was shown that the reflection coefficient is zero for a cylinder in the infinite lower layer of a two-layer fluid, while zero reflection is not observed for a cylinder in the upper layer. The certain recursive relations were proposed by Khabakhpasheva and Sturova [2] to analyze the diffraction of the internal waves by a submerged circular cylinder in a uniform current of a two-layer fluid of infinite depth. Furthermore, Linton and Cadby [3] considered three-dimensional problems with the oblique waves and found that the incident energy is reflected at some isolated frequencies whether a cylinder is in the upper or lower fluid layer.

With the development of the researches in the polar regions, the interest gradually ranges from pure gravity waves to flexural–gravity waves through ice sheets [4–8]. The ice sheets are often modeled as thin plates floating on the ocean without draft, which are also used for modeling very large floating structures (VLFSs) in ocean engineering [9]. Das and Mandal [10] extended the work of Linton and Cadby [3] by considering an ice cover on the upper fluid and obtained similar conclusions for the reflection coefficients. The interaction of flexural–gravity waves with a vertical rigid wall was studied by Brocklehurst et al. [11] with the method of the integral transform. On the basis of this, Bhattacharjee and Guedes Soares [12] considered the rigid wall to be either fixed or harmonically oscillating with a constant horizontal displacement. Mohapatra and Bora [13] discussed the exciting forces for vertical and horizontal directions due to the interaction of water waves with a submerged sphere in an ice-covered two-layer fluid of finite depth. Lin and Lu [14] investigated the hydroelastic responses of a

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semi-infinite elastic plate on a two-layer fluid due to obliquely incident waves, and derived one critical angle for the incident waves of the surface wave mode and three critical angles for the incident waves of the interfacial wave mode, which are related to the existence of the propagating waves.

The interaction between pure gravity waves and a vertical cylinder in the homogeneous fluid is well studied. Sabuncu and Calisal [15] gave the hydrodynamic coefficients of vertical circular cylinders for different depth-to-radius and draft-to-radius ratios. With the method of the eigenfunction expansion, Yeung [16] calculated the added mass and damping of a vertical floating cylinder for different motions. Rahman and Bhatta [17] extended the method to the horizontal oscillations of a large surface-piercing and bottom-mounted vertical circular cylinder. Recently, some other situations for the wave–cylinder interaction are considered. You et al. [18] investigated the radiation and diffraction of water waves by a bottom-mounted circular cylinder in a two-layer fluid. Shi et al. [19] further considered the floating circular cylinder and found that the density stratification has obvious effects on the interfacial wave mode over a certain range of frequency. In addition, several scholars considered the flexural–gravity waves interacting with the vertical cylinder. Malenica and Korobkin [20] analyzed the water-wave interaction with a vertical cylinder frozen into a circular finite ice floe. The ice of infinite extent was studied by Brocklehurst et al. [21] with the method of integral transforms.

This paper herein represents an extension of the investigations of You et al. [18] and Malenica and Korobkin [20] to the case that a bottom-mounted circular cylinder is clamped into a circular finite plate in a two-layer fluid. For the pure gravity waves in the two-layer fluid, the distinct eigenfunctions with different wave numbers are orthogonal to one another, which indicates that only the propagating wave modes matter and the evanescent wave modes vanish completely. However, it is not the case for the flexural–gravity waves since the eigenfunctions are not orthogonal mutually. Both the propagating wave modes and the evanescent wave modes are considerable. We modify the method of Malenica and Korobkin [20] for a single-layer fluid by directly applying the inner product on the lateral boundary condition around the rigid cylinder regardless of a vast amount of calculation to verify the number of the independent eigenfunctions, and then introduce this modification to the study for a case with a two-layer fluid.

The mathematical model involving the governing equations and the boundary conditions is formulated in Section 2. In Section 3, we present the dispersion relations for the pure gravity and flexural–gravity waves, and then express the velocity potentials with the appropriate Bessel functions obtained via the method of the angular eigenfunction expansion. With the aid of a newly defined inner product for a two-layer fluid, we can derive a closed system of simultaneous linear equations. The results of numerical calculations and the discussion are shown in Section 4. The optimum truncation values of the two series, the horizontal forces and the moments are figured out under different incident waves. Finally, conclusions are made in Section 5.

2. Mathematical formulation

We consider the diffraction of incident surface and interfacial waves by a vertical rigid circular cylinder with the radius a, which is fixed at the center of a circular elastic plate with the radius R, in a two-layer fluid of finite depth, as shown in Fig. 1. The bottom of the cylinder is fixed on the rigid seabed and the top pierces the plate. The circular plate, which is free at its ends, centers at the origin of coordinates. Cylindrical coordinates (r, θ, z) are chosen such that the plane $z = 0$ coincides with the undisturbed water surface. The z-axis points vertically upwards with $z = -h_1$ as the interface between the two fluids and $z = -H$ as the flat bottom. The upper fluid layer, with the constant density ρ_1, occupies the region $-h_1 < z < 0$, while the lower fluid layer, with the constant density ρ_2, occupies the region $-H < z < -h_1$. With the assumptions that the upper and lower fluids are inviscid and incompressible and that the motion is irrotational, the problem may be described by the potential flow theory. The whole flow domain is mathematically divided into two parts, an open water region ($R < r < \infty$) and a plate-covered region ($a \le r \le R$).

In the case of the simply time-harmonic incident waves with a given angular frequency ω, we write, via separating the time factor, the velocity potential as $\mathrm{Re}[\phi_m(r, \theta, z)\mathrm{e}^{-\mathrm{i}\omega t}]$ with $m = 1, 2$, the surface elevation as $\mathrm{Re}[\zeta(r, \theta)\mathrm{e}^{-\mathrm{i}\omega t}]$ and the interfacial elevation as $\mathrm{Re}[\eta(r, \theta)\mathrm{e}^{-\mathrm{i}\omega t}]$, where $\phi_m(r, \theta, z)$ is the spatial velocity potential in the upper ($m = 1$) and lower ($m = 2$) fluids; $\zeta(r, \theta)$ and $\eta(r, \theta)$ are the spatial elevations on the surface ($z = 0$) and the interface ($z = -h$), respectively; and t is the time variable. The spatial velocity potential $\phi_m(r, \theta, z)$ in the whole fluid domain satisfies the Laplace equation

$$\left(\nabla^2 + \frac{\partial^2}{\partial z^2}\right)\phi_m = 0, \quad (a \le r < \infty,\ -H < z < 0), \tag{1}$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{\partial}{r\partial r} + \frac{\partial^2}{r^2\partial\theta^2}$ and $m = 1, 2$.

Within the framework of the linear theory for small-amplitude waves, the combination of the kinematic and dynamic boundary conditions on the free surface may be written as

$$-\omega^2\phi_1 + g\frac{\partial\phi_1}{\partial z} = 0, \quad (R < r < \infty,\ z = 0), \tag{2}$$

for the open water region, where g is the gravitational acceleration. If there is no cavitation between the fluid and the elastic plate, the surface pressure of the upper fluid is external load of the plate. Thus the combined boundary condition on the fluid–plate interface is mathematically represented as

$$-\rho_1\omega^2\phi_1 + \left(D\nabla^4 - m_e\omega^2 + \rho_1 g\right)\frac{\partial\phi_1}{\partial z} = 0, \quad (a \le r \le R,\ z = 0), \tag{3}$$

for the plate-covered region, where D is the flexural rigidity of the plate, which is expressed, in terms of Young's modulus E, Poisson's ratio ν and the thickness of the plate d, as $D = Ed^3/12(1-\nu^2)$, and $m_e = \rho_e d$ is the mass per unit area of the plate. From the kinematic and dynamic boundary conditions and the continuities of the velocity and the pressure on the interface between the upper and lower fluids, we have

$$\frac{\partial\phi_1}{\partial z} = \frac{\partial\phi_2}{\partial z} = -\mathrm{i}\omega\eta, \quad (a \le r < \infty,\ z = -h_1), \tag{4}$$

$$\gamma\left(\frac{\omega^2}{g}\phi_1 - \frac{\partial\phi_1}{\partial z}\right) = \frac{\omega^2}{g}\phi_2 - \frac{\partial\phi_2}{\partial z}, \quad (a \le r < \infty,\ z = -h_1), \tag{5}$$

where $\gamma = \rho_1/\rho_2$ and $0 < \gamma < 1$. In addition, the boundary condition on the flat rigid bottom reads

$$\frac{\partial\phi_2}{\partial z} = 0, \quad (a \le r < \infty,\ z = -H). \tag{6}$$

The circular plate floats on the fluid with the free-edge condition, which is free of forces and bending moments and can be written as

$$\left[\frac{\partial}{\partial r^2} + \nu\left(\frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\right]\zeta = 0, \quad (r = R,\ z = 0), \tag{7}$$

$$\left[\frac{\partial}{\partial r}\nabla^2 + \frac{1-\nu}{r^2}\left(\frac{\partial}{\partial r} - \frac{1}{r}\right)\frac{\partial^2}{\partial\theta^2}\right]\zeta = 0, \quad (r = R,\ z = 0). \tag{8}$$

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