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Convective instability in layered sloping warm-water aquifers

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ABSTRACT

In a recent paper (McKibbin et al., 2011) some questions were posed about fluid and heat flows in a stratified groundwater aquifer with a small slope, subject to a perpendicular temperature gradient. The strength of shear flows in the direction of the maximum slope and the associated convected heat flux were quantified. This paper provides a method to quantify the stability of such flows and the shape and amplitude of convective rolls that may form when the critical Rayleigh number is exceeded. The associated issue of how the mean flow-path of a soluble species introduced into the aquifer is affected by the convective rolls, is also considered. The models formulated are for buoyancy-driven fluid flow in long, sloping warm-water aquifers with both smoothly- and discretely-layered structures.

1. Introduction

Thermally-driven convection in porous media occurs in a wide variety of geophysical and industrial settings. It has been extensively studied and modelled both for particular applications and as a more generic phenomenon, and is comprehensively reviewed by Nield & Bejan [1]. Early work [2,3] that investigated the criterion for onset of convection in a homogeneous isotropic uniform layer heated from below has been extended in many ways since. In particular, the spatial structure of the permeable system has been allowed to vary in later models – critical conditions for anisotropic horizontal layers were found [4], the stability of horizontally-layered porous slabs was investigated in McKibbin & O'Sullivan [5,6], and the anisotropic analogue of a layered system was studied in [7] – such work was reviewed in [8]. The motionless equilibrium state of a saturated permeable layer may be stable to thermal gradients in special cases (horizontal layer, uniform temperature horizontal boundaries, small enough Rayleigh number, etc.); however, any thermal non-uniformity on the boundaries or heat sources, or any variation in thermal conductivity that is not purely horizontal or vertical, usually provides conditions for convective motion. In particular, non-horizontal heated boundaries immediately provide thermal buoyancy forces that drive the fluid.

Convection in sloping permeable layers has been studied by several authors. Investigations of multi-layered systems subjected to uniform vertical salinity and/or temperature gradients were reported in [9,10]; the systems were considered to be unaffected by distant boundaries. Provided the system Rayleigh number Ra is not too large in a homogeneous system ($\mathrm{Ra}\cos\alpha < 4\pi^2$, where α is the layer slope angle), a steady stable unicellular convection cell may be established [9,11–13], and the flow far from the ends of a system with a small thickness-to-length ratio will be parallel to the upper and lower boundaries. Numerical and experimental investigations of thermally-induced convection in tilted fractures filled with a homogeneous porous material within an otherwise impermeable slab were reported by Medina et al. [14]; the temperature and fluid flow profiles were found to be approximately linear across the fracture.

Quantification of such steady convective motion induced by temperature differences across a sloping porous slab of finite thickness was provided in [15]. The heat flow is dominated by conduction, but a small-scale convective motion is induced; this is additional to any net through-flow that may present due to a small dynamic pressure gradient along the system. Under the conditions described above, the flow is parallel to the sloping boundaries. A model was formulated for the case where the aquifer system has a layered structure, smooth or discrete, due to geological bedding. This paper provides a brief review of that study, and continues with a formulation of the equations that may be used to quantify the system's state when it is liable to instability. Examples are used to illustrate the method but, because the parameters are so numerous, only a small selection can be shown.

First, the equations that describe the movement of the fluid and thermal energy are summarized. The steady-state fluid flow and temperature profiles due to natural convection for a sloping system reported in [15] are repeated briefly here.

The aim is to use the natural bedded structure of such geological systems to advantage. The results of this model should be applicable to analyzing fluid fluxes and temperature profiles occurring in sloping aquifers. It is worth noting that most aquifers have some degree of tilt caused by crustal movement after a sequence of alluvial layers is deposited. Use of this model may enable quantification of resulting natural flows.

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Nomenclature

(SI Units given where appropriate)

a	Convective roll amplitude (–)
c	Specific heat (J kg^{-1} K^{-1})
d	Sub-layer thickness (m)
g	Gravitational acceleration (m s^{-2})
H	Total system thickness (m)
ℓ, m	Wavenumbers
k	Thermal conductivity (W m^{-1} K^{-1})
L	Cell-width (–)
K	Permeability (m^2)
n	Layer number
N	Total number of sub-layers
p	Fluid pressure (Pa)
q	Specific flux (m^2 s^{-1})
Q	Dimensionless net volume flux (–)
Ra	Rayleigh number (–)
s	Growth rate (–)
t	Time (s)
T	Temperature (K)
$\underset{\sim}{u}$	Fluid specific volume flux (m s^{-1})
u, v, w	Components of $\underset{\sim}{u}$ (m s^{-1})
W	Flow speed function (–)
x, y, z	Spatial coordinates (m)

Greek symbols

α	Slope angle (rad)
β	Thermal expansivity (K^{-1})
δ	Scaled sub-layer thickness (–)
ζ, σ	Continuous variables
η	Thermal conductivity anisotropy (–)
θ	Temperature function (–)
κ	Scaled permeability (–)
μ	Dynamic viscosity (kg m^{-1} s^{-1})
ν	Kinematic viscosity (m^2 s^{-1})
ξ	Permeability anisotropy (–)
ρ	Density (kg m^{-3})
ϕ	Porosity (–)
χ	Scaled thermal conductivity (–)
ψ	Stream function (–)
ΔT	Temperature difference (K)

Subscripts

$\perp$	Perpendicular to bedding plane
$=$	Parallel to bedding plane
0	Datum value at axes origin
a	Datum value at upper boundary
crit	Critical value
f	Fluid
i, j, r	Layer numbers
m	Matrix-fluid mixture value
P	At constant pressure
s	Solid matrix
SS	Steady state
W	Matrix operator
α	Matrix operator
θ	Matrix operator
K	Matrix operator
min	Corresponding to minimum value
x, y	Axis directions

Superscripts

$\hat{}$	(Hat) non-dimensionalized quantity
$\bar{}$	(Overbar) weighted average
$'$	Perturbation value
$*$	Special value (as defined in text)
a	Along slope
c	Cross slope
temp	Temporary value

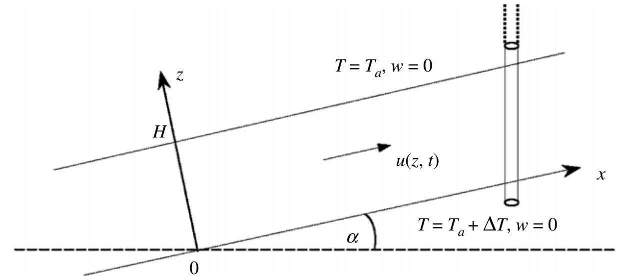

Fig. 1. Schematic cross-section of a sloping aquifer with impervious and isothermal boundaries. The *x*-axis is aligned in the direction of maximum slope and the *z*-axis is perpendicular to it. The temperature of the lower boundary is higher than that of the upper one.

2. The equations for fluid and heat flow

Consider convection of a fluid (considered here to be a liquid) within a sloping porous layer of uniform total thickness H, bounded above and below by plane impervious surfaces (a schematic is shown in Fig. 1). The upper boundary is maintained at temperature T_a while the bottom boundary is kept at the higher temperature $T_a + \Delta T$. Cartesian coordinates (x, y, z) are aligned so that the x-axis is parallel to the base of the sloping layer and positive in the up-slope direction (angle $\alpha \geq 0$ to the horizontal), while the z-axis is perpendicular to the base of the layer. The gravitational acceleration vector is then given by $\underset{\sim}{g} = (-g \sin\alpha, 0, -g\cos\alpha)$. The layer lies between the hotter bottom boundary at $z = 0$ and the cooler top surface at $z = H$.

The matrix parameters may vary through the thickness of the layer. The equations describing conservation of mass, momentum and energy of the fluid within the layer are, invoking the Boussinesq assumption and neglecting inertia in the momentum balance (see [1], for example):

$$\nabla \cdot \underset{\sim}{u} = 0$$
$$\underset{\sim}{u} = \frac{K}{\mu_f}(-\nabla p + \rho_f \underset{\sim}{g}) \quad (1)$$
$$(\rho c)_m \frac{\partial T}{\partial t} = -\nabla \cdot [(\rho c_P)_f T \underset{\sim}{u} - k_m \nabla T].$$

Here, $\underset{\sim}{u} = (u(x, y, z, t), v(x, y, z, t), w(x, y, z, t))$ ((m^3 s^{-1}) m^{-2} = m s^{-1}) is the specific fluid volume flux (usually termed the Darcy velocity), $K(z)$ (m^2) is the matrix permeability (assumed to be locally isotropic), $T(x, y, z, t)$ (K) is the temperature, $\rho_f(T)$ (kg m^{-3}) is the fluid density, $\mu_f(T)$ (kg m^{-1} s^{-1}) is the dynamic viscosity of the fluid and $p(x, y, z, t)$ (Pa = kg m^{-1} s^{-2}) is the fluid pressure.

The specific heat content $(\rho c)_m$ and the thermal conductivity k_m of the fluid-saturated medium (a matrix-fluid "mixture"), both subscripted m, may be found by suitably-weighted combinations of the solid matrix (subscript s) and fluid parameters (subscript f). For example [1]:

$$(\rho c)_m = (1 - \phi)(\rho c)_s + \phi(\rho c_p)_f$$
$$k_m = (1 - \phi)k_s + \phi k_f.$$

Here, $c_p(T)$ (J kg^{-1} K^{-1}) is the specific heat of the fluid, ϕ (dimensionless) is the matrix porosity and $k_m(z)$ (W m^{-1} K^{-1}) is the (locally isotropic) thermal conductivity of the fluid-saturated porous medium.

The density variation (due to thermal expansion) of the fluid is neglected except in the momentum equation (Darcy's Law), where it is approximated by $\rho_f = \rho_a[1 - \beta_a(T - T_a)]$ with $\rho_a = \rho_f(T_a)$ (kg m^{-3}), where T_a is the (reference) upper boundary temperature, and β_a (K^{-1}) is the fluid thermal expansivity at $T = T_a$; elsewhere, $\rho_f = \rho_a$. Likewise, the fluid's dynamic viscosity, $\mu_a = \mu_f(T_a)$ and