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The lift-up effect: The linear mechanism behind transition and turbulence in shear flows

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ABSTRACT

The formation and amplification of streamwise velocity perturbations induced by cross-stream disturbances is ubiquitous in shear flows. This disturbance growth mechanism, so neatly identified by Ellingsen and Palm in 1975, is a key process in transition to turbulence and self-sustained turbulence. In this review, we first present the original derivation and early studies and then discuss the non-modal growth of streaks, the result of the lift-up process, in transitional and turbulent shear flows. In the second part, the effects on the lift-up process of additives in the fluid and of a second phase are discussed and new results presented with emphasis on particle-laden shear flows. For all cases considered, we see the lift-up process to be a very robust process, always present as a first step in subcritical transition.

1. Introduction

1.1. *"Stability of linear flow"*

This is the title of a research note in Physics of Fluids of less than two pages published in 1975 by Ellingsen and Palm. In this work, the authors identify a linear mechanism responsible for the amplification of fluctuations in shear flows. In their own words, *a finite disturbance independent of the streamwise coordinate may lead to instability of linear flow, even though the basic velocity does not possess any inflection point.* This mechanism, later denoted as the lift-up effect, is a key process in the laminar–turbulent transition in shear flows and in fully developed turbulence, as will be discussed in this review.

At the time of their note, the main general results for the linear stability of shear flows were Rayleigh, Fjørtoft and Howard criteria [2]. Rayleigh's criterion states that a necessary condition for the instability of a parallel shear flow is that the basic velocity profile has an inflection point [3]. Later Fjørtoft [4] showed that the vorticity needs to have a maximum at the inflection point. Howard [5] proved that the complex phase velocity of an exponential wave must lie within a semi-circle having a diameter equal to the difference between the largest and the smallest velocity of the parallel base flow. These theorems are valid in an inviscid and not stratified fluid and were obtained by considering two-dimensional infinitesimal perturbations, i.e. directly from the linearized Rayleigh equation for the stability of a parallel shear flow. Squire's theorem [128] states that two-dimensional disturbances are the first to become unstable in parallel shear flows and thus they determine the critical Reynolds number; this had restricted the stability analyses to two-dimensional normal modes (exponential growth or decay of periodic waves).

Ellingsen and Palm's fundamental contribution is to show that three-dimensional disturbances may lead to an instability other than modal, independent of the existence of an inflection point. They note how this instability can be responsible for transition to turbulence and acknowledge previous suggestions by Høiland (referring to some unspecified lecture notes), who, however, *did not draw full conclusions from his idea.* Indeed, this new mechanism is able to explain transition in subcritical conditions or in stable flows as in the case of pipe flow [6].

We will shortly outline here the original derivations and denote a parallel velocity profile as $\mathbf{U} = (U, V, W) = (U(y), 0, 0)$ where U is the streamwise velocity component and y and z the cross-stream coordinates. Considering an inviscid, incompressible and not stratified flow bounded by two parallel planes and a disturbance independent of the streamwise coordinate x, the equation for the streamwise component of the momentum and for the streamwise vorticity component reduce to

$$\frac{Du}{Dt} = 0; \qquad \frac{D\xi}{Dt} = 0. \tag{1}$$

Introducing a streamfunction Ψ for the cross-stream components,

$$v = \frac{\partial \Psi}{\partial z}, \qquad w = -\frac{\partial \Psi}{\partial y};$$

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and linearizing one obtains

$$\frac{\partial u}{\partial t} + v\frac{dU}{dy} = 0 \tag{2}$$

for the streamwise disturbance velocity and

$$\frac{\partial}{\partial t}\nabla_1^2 \Psi = 0 \tag{3}$$

for the cross-stream flow, where ∇_1^2 is the two-dimensional Laplacian. From Eq. (3), we see that the cross-stream velocity components are independent of time, i.e. a streamwise independent perturbation v will not grow or decay in an inviscid flow. Eq. (2) can be integrated

$$u = u(0) - v\frac{dU}{dy}t \tag{4}$$

to show that the perturbation u grows linearly in time, from which also the name of algebraic inviscid instability. It is hence shown that any shear flow $U(y)$ is unstable to streamwise independent disturbances in the cross-stream velocity components.

This first part of the original paper, based on a linear analysis, is probably the most known and commonly used as a reference for the optimal transient growth of x-independent perturbations in viscous flows; see Schmid and Henningson [7] and discussion below. Indeed, we will see that infinitely long streamwise vortices are the most dangerous initial conditions in shear flows: they lead to the formation of streamwise streaks, elongated regions of positive and negative streamwise velocity, by redistributing streamwise momentum across the shear layer.

Ellingsen and Palm, in addition, show that Eq. (1) can be solved also for finite-amplitude perturbations. The conservation of streamwise vorticity can be re-written as

$$\frac{\partial}{\partial t}\nabla_1^2 \Psi + \frac{\partial \Psi}{\partial z}\frac{\partial}{\partial y}\nabla_1^2 \Psi - \frac{\partial \Psi}{\partial y}\frac{\partial}{\partial z}\nabla_1^2 \Psi = 0, \tag{5}$$

which admits solution of the form

$$\nabla_1^2 \Psi = f(\Psi), \qquad \frac{\partial \Psi}{\partial t} = 0, \tag{6}$$

with f an arbitrary function. If f is a linear function, the cross-stream motion is represented by a set of closed streamlines. The conservation of momentum in the streamwise direction then implies that velocity u is conserved during the motion along these closed streamlines. *A fluid particle in its orbit in the x–y plane will, therefore, have a u velocity equal to the value of the basic flow at the initial position of that particle.* This value will be different from the initial local value of u, the more the larger the vertical particle displacement in a homogeneous shear. As the period can be different along different streamlines, the motion is aperiodic with a complete redistribution of streamwise momentum. This is independent of the initial disturbance amplitude and may lead to large velocity gradients that, in turn, can support new instabilities: *It is possible of course that the developed motion is unstable. Owing to the large vorticity concentrations this indeed seems very likely so that the motion already discussed is valid only for a short span of time*. This is indeed what happens in the case of secondary streak instability, where an inflectional type of instability develops on the regions of largest vorticity induced by streamwise elongated perturbations [8–10]. However, the vertical displacement of fluid particles by the cross-stream momentum is not observed only for a short time, as cautiously stated by Ellingsen and Palm. This is a key ingredient not only for the breakdown to turbulence but also in the dynamics of wall-bounded turbulence, as we will show in this review.

Ellingsen and Palm conclude that, despite their analysis is limited to the case of streamwise independent disturbances, the equations are valid also when the base flow has an angle with the x-direction, $\mathbf{U} = (U(y), 0, W(y))$. For small angles, the physical mechanisms at play (cross-stream displacement of fluid particles that retain their horizontal momentum) are the same. For larger angles, however, the variations of the streamwise velocity u are much smaller as the disturbance field has a component in the x-direction. This seminal paper ends by stating that *by same reasoning we obtain the result that an inviscid channel flow is always unstable for perturbations independent of the streamwise coordinate.* This explains the first stage of the subcritical transition to turbulence in pipe flow, a problem that puzzled scientists for over a century [11,12]. Indeed the lift-up effect becomes the only responsible for disturbance energy growth when no other modal instabilities are present.

1.2. Early inviscid studies

In a review paper from 1969 about shear-flow turbulence, Phillips reports a previous analysis by Moffatt aiming to explore whether a disturbance can maintain itself by interactions with the mean shear [13,14]. Considering the interactions between middle-size eddies and a uniform shear flow, $U = Sy$, Moffatt determined solutions of the linearized Navier–Stokes equations for three-periodic velocity perturbations and pressure

$$u_i = A_i(t)\exp[i(\mathbf{k}(t)\cdot\mathbf{x})]; \qquad p = \pi(t)\exp[i(\mathbf{k}(t)\cdot\mathbf{x})] \tag{7}$$

with wavenumber

$$\mathbf{k}(t) = (k_x, k_y, k_z) = [k_x(0), k_y(0) - Stk_x(0), k_z(0)]. \tag{8}$$

The latter expression indicates that each Fourier component is tilted by the shear, where the lines of constant phase move closer together and rotate until they become asymptotically parallel to the planes defined by a constant value of coordinate y. Moffatt also derives a dynamical equation of the velocity amplitudes A_i and shows that for streamwise independent modes ($k_x = 0$) the solution can be written as

$$A_x(t) = A_x(0) - StA_y(0); \qquad A_y(t) = A_y(0) : A_z(t) = A_z(0). \tag{9}$$

The streamwise velocity perturbation grows linearly in time if the initial disturbance has a non-zero component in the wall-normal direction, as shown by Ellingsen and Palm for a bounded shear flow and any general disturbance shape in the linear and nonlinear regimes. A superposition of periodic disturbances evolves towards a series of horizontal structures with vanishing cross-stream velocity components and vanishingly small scales in the y-direction, something which would accelerate viscous dissipation. Moffatt calculated the Reynolds stress associated to these structures and showed that the flow will asymptotically tend to one dominated by large-scale structures, independent of the x-coordinate. Phillips notes in his review that the disturbance amplification computed by Moffatt corresponds to cross-stream displacement of fluid particles retaining their original streamwise momentum but this cannot explain how turbulence is sustained although there is abundant evidence of the presence of such elongated structures in wall turbulence. Studies of homogeneous-shear turbulence shed anyway light on the energy transfer among Fourier modes represented by the tilting of the disturbance and the lifting of the elongated streaks observed in turbulence.

As mentioned above, the historical basis for the paper by Ellingsen and Palm was the work on hydrodynamic stability by Palm's mentor Einar Høiland. Remarkably, Palm's paper was his last contribution to the stability analysis of homogeneous fluids. Nobody in Norway followed up this research. This was however continued in Sweden, due to the influence of Palm's friend, Mårten Landahl. Few years later, Landahl [15] studied the dynamics of shear flow turbulence and the burst events, always associated to a low-speed streak lifting from the surface and forming locally a

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