ARTICLE IN PRESS

European Journal of Mechanics B/Fluids ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Third-order effects in wave–body interaction

B. Molin [a,*], O. Kimmoun [a], F. Remy [a], I.K. Chatjigeorgiou [b]

[a] *Aix Marseille Université, CNRS, Centrale Marseille, IRPHE UMR 7342, 13451 Marseille cedex 20, France*

[b] *National Technical University of Athens (NTUA), Greece*

ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Wave–body interaction
Potential flow theory
Third-order effects
Run-up
Boussinesq model

ABSTRACT

A review is presented of about 10 years research work on phenomena resulting from third-order interactions between an incoming wave system and the reflected wave system by a structure. The most striking manifestations are large run-ups that can be observed locally on the weather side. Two types of numerical potential flow models are described, one seeking for a steady-state solution in the frequency domain, and the other a fully nonlinear numerical wavetank based on extended Boussinesq equations. Numerous experimental investigations are described, with vertical walls and arrays of vertical cylinders. Computational results compare favorably with the measurements. Important issues such as the size of the interaction area and possible chaotic behavior are finally discussed.

1. Introduction

The purpose of the present paper is to review and synthesize a research work carried out over a time span of ten years, on some beforehand little known, or not clearly understood, phenomena. These phenomena result from third-order interactions between the incoming wave system and the reflected/radiated wave field from the body. They do not result into high-frequency loads or related phenomena taking place, in regular waves, at three times the frequency of the incoming waves (known as ringing loads—see, e.g., [1]), but into alterations of first-harmonic quantities. Via these third-order interactions the reflected wave field "slows down" locally the incoming wave system (alike a shoal would do), inducing focusing effects. The most striking manifestations are large run-ups at some places along coastal walls or ship hulls whereby the local free surface elevations can reach 4 or 5 times the amplitude of the incoming waves, at variance with the predictions of linear theories.

As a matter of fact, our first encounter with the problem was while carrying out model tests, at CEHIPAR (*Canal de Experiencias Hidrodinamicas de El Pardo*), in 2001, on the resonant roll response of rectangular barges in beam seas: large run-ups took place at midship, on the weather side, to the point that the model started shipping water and the test program had to be revised to milder conditions. According to linearized potential flow theory the free surface elevation along the hull, at the considered wave frequencies, should have remained nearly rectilinear, far from the visual observations. In irregular waves the same phenomenon was observed with the relative free surface elevations at midship being two or three times higher than expected.

The CEHIPAR test results were confirmed by a subsequent experimental campaign, in the BGO-FIRST wave tank, located at la Seyne-sur-mer. Fig. 1 shows the observed wave run-up at mid-hull of a barge model, with length 1.7 m, beam 0.6 m and draft 0.12 m, in regular waves of period 0.9 s and amplitude 2.5 cm ($H/L \simeq 4\%$). By that time we had suspected that third-order interactions between incoming and reflected wave fields were at hand and the following campaign was devised in order to confirm this speculation. The experimental model was reduced to a vertical plate, 1.2 m long, stuck against one of the walls of the tank, at 19 m from the wavemaker. By optical symmetry this set-up was equivalent to a 2.4 m long plate in the center of a 32 m wide tank. Six gauges were set along the weather side of the plate, as shown in the photograph (Fig. 2). Tests were run in regular waves only, with wavelengths ranging from 1.2 m up to 3 m, and steepnesses $2A_I/L$ from 2% up to 6% (A_I being the amplitude and L the wavelength). Most of the test results and analysis are reported in [2]. Fig. 3, taken from this reference, shows the measured free surface elevations at the gauge nearest to the plate–wall corner, in regular waves of 1.2 m wavelength and steepnesses of 2% (top), 3%, 4%, 5% and 6% (bottom). The elevations have been normalized by the amplitude of the incoming waves (as measured during wave calibrations). If things were linear one should have obtained the same time series 5 times. It can be seen that only in the lowest steepness case a steady-state is quickly obtained with a periodic signal oscillating between −2 and +2, as expected from linear theory. From the 3% steepness the signal amplitude appears to increase with time, at a rate that can be checked to be proportional to the square of the steepness. In the 4% and 5% steepness cases, it looks like a steady-state has been attained after many cycles, with the signal amplitude being finally around twice

* Corresponding author. Tel.: +33 491054641.
E-mail address: bernard.molin@centrale-marseille.fr (B. Molin).

0997-7546/$ – see front matter © 2013 Elsevier Masson SAS. All rights reserved.
http://dx.doi.org/10.1016/j.euromechflu.2013.11.009

Please cite this article in press as: B. Molin, et al., Third-order effects in wave–body interaction, European Journal of Mechanics B/Fluids (2014), http://dx.doi.org/10.1016/j.euromechflu.2013.11.009

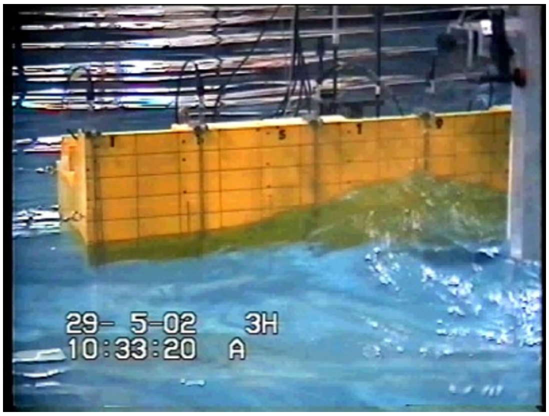

Fig. 1. Run-up on a barge model at BGO-FIRST.

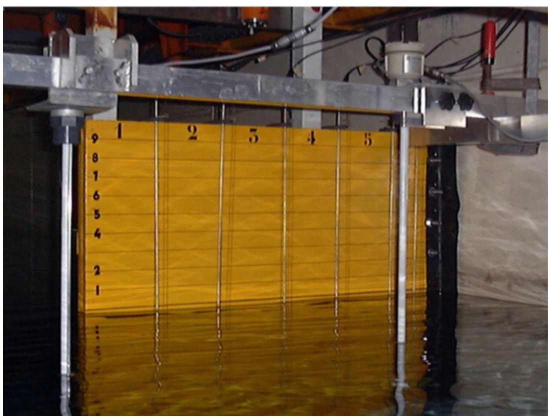

Fig. 2. The 1.2 m long plate at BGO-FIRST.

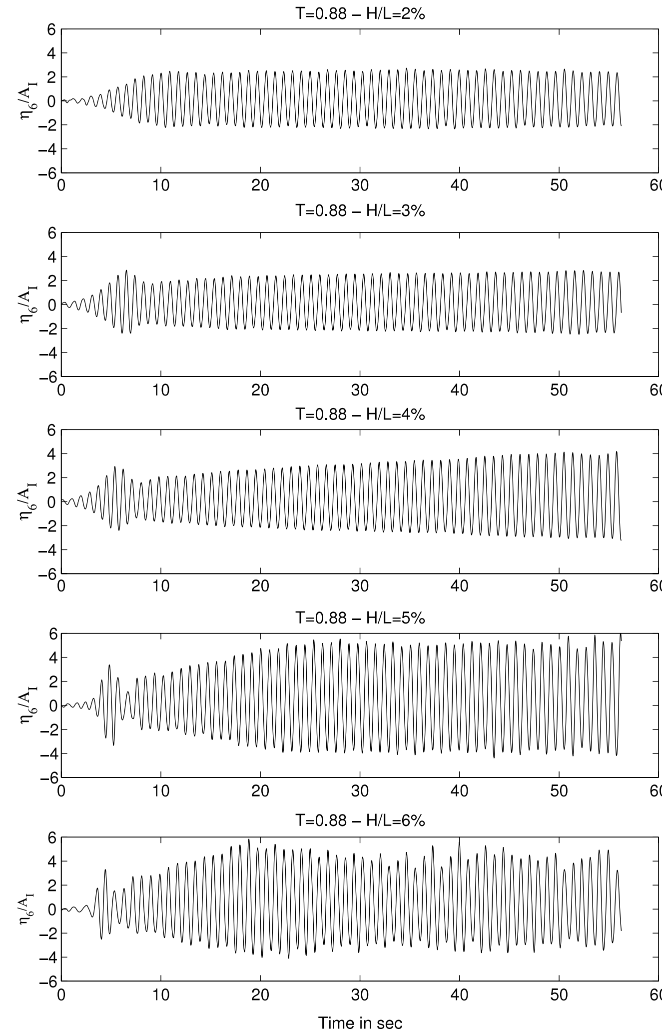

Fig. 3. Time series of the normalized free surface elevation at the plate–wall corner. Wave period: 0.88 s. Steepnesses $2A_I/L$ from 2% (top) to 6% (bottom).

the expected value. At 6% steepness the signal becomes somewhat erratic after the first ramp: this is associated with local breaking taking place in the basin.

That the rate of increase of the signals is proportional to the steepness squared suggests third-order nonlinearities. This is confirmed by the fact that the free surface elevations, as measured at the plate, gradually lag in time as referred to the wave elevation at a gauge by the other side of the tank: the rate of increase of the time lag is also proportional to the steepness squared (see Figs. 8 and 11 in [2]). That the reflected wave system "slows down" the incoming waves as they progress toward the plate is expected from third-order theories applied to the case of plane waves, as shown more than 50 years ago by Longuet-Higgins and Phillips [3].

The organization of this paper is as follows. In the next section we briefly recall the third-order theory of Longuet-Higgins and Phillips [3] who tackled the case of two plane waves, of different frequencies and directions of propagation, in infinite depth. Then we present the numerical models that were developed in order to reproduce, numerically, the observed phenomena. One first family of models looks for a steady-state solution, in the frequency domain, through iterations. A second type is a fully nonlinear numerical wavetank, based on extended Boussinesq equations in potential form. Illustrations of the performances of these models are given, taken from [4] in the case of a 3 m long plate also tested at BGO-FIRST. As yet unpublished results from tests on an "infinite" wall with a narrow gap, and on a square array of vertical cylinders, together with computational results, are then presented. Finally some important issues are discussed, like the sensitivity of the computed/observed phenomena to the size of the numerical/physical domain ahead of the structure, and the impossibility, in some cases, to reach a steady-state.

2. Third-order interactions between plane waves

In the past fifty years, since the early papers by Phillips and by Longuet-Higgins (see [5]), much work has been done on third-order interactions between plane gravity waves. Considering a basic set of 4 components, it is known that when their wave number vectors and frequencies are linked by the relationships

$$\overrightarrow{k_1} + \overrightarrow{k_2} \simeq \overrightarrow{k_3} + \overrightarrow{k_4}, \tag{1}$$

$$\omega_1 + \omega_2 \simeq \omega_3 + \omega_4, \tag{2}$$

interaction will occur, leading to mutual modifications of phases and amplitudes. A review can be found, e.g., in [6].

In the case when there are only two components, that is $\overrightarrow{k_1} \equiv \overrightarrow{k_3}$, $\overrightarrow{k_2} \equiv \overrightarrow{k_4}$, no energy exchanges take place and only the phase velocities are affected.

Longuet-Higgins and Phillips [3] tackled the case of two plane waves in infinite depth. In the particular case when the two frequencies are equal, from their analysis, one obtains that the wave number of the first component, of amplitude A_1, is modified by a quantity $k_1^{(2)}$ given by

$$k_1^{(2)} = k^3 A_2^2 f(\beta) + \frac{1}{2} k^3 A_1^2 f(0) = k^3 A_2^2 f(\beta) - k^3 A_1^2, \tag{3}$$