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Cloaking a circular cylinder in water waves

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ABSTRACT

In the diffraction of water waves by fixed bodies, the scattered waves propagate outward in the far field and attenuate with increasing distance from the structure. 'Cloaking' refers to the reduction in amplitude or complete elimination of the scattered waves. The possibility of cloaking is of both scientific and practical interests.

Cloaking is considered here for a circular cylinder on the free surface, surrounded by one or more additional bodies. Linearized time-harmonic motion is assumed. A numerical procedure is used to optimize the geometry of the surrounding bodies, so as to minimize the energy of the scattered waves. Values of the scattered energy are achieved which are practically zero at a specific wavenumber, within the estimated numerical accuracy. This provides tentative support for the existence of perfect cloaking, and conclusive evidence that structures can be designed to have very small values of the mean drift force.

Dedicated to the memory of Enok Palm, an inspiring colleague and friend

1. Introduction

In the three-dimensional diffraction problem, where plane waves are incident upon a fixed structure, scattered waves generally exist in the far field. The word 'cloaking' is used in various fields of wave motion to refer to the reduction in amplitude or complete elimination of the scattered waves. This is achieved by modifying the shape of the structure or the properties of the surrounding medium. 'Perfect cloaking' refers to the condition where there are no scattered waves in any direction. The possibility of perfect cloaking in the diffraction of water waves is of scientific interest, since it is not known if this condition can be achieved with a structure of non-zero volume on or near the free surface.

Cloaking may also have practical applications in the design of offshore structures, particularly with respect to the mean drift force. When scattering occurs the time-averaged second-order pressure exerts a steady drift force on the structure, in the direction of propagation of the incident waves. This drift force can be related by momentum conservation to the amplitude of the scattered waves. Thus the mean drift force is zero if there are no scattered waves.

Energy is transported by the scattered waves as they propagate outward on the free surface. The total scattered energy is defined here as the integral of the rate of energy flux across a control surface surrounding the structure. In an ideal fluid the mean rate of energy flux is constant, independent of the control surface. Since the energy is proportional to the square of the wave amplitude it follows that the amplitude is proportional to the inverse square-root of the radius. If there are no scattered waves the scattered energy is equal to zero. Thus the scattered energy is an appropriate measure of cloaking, analogous to the scattering cross-section in other fields.

Cloaking a bottom-mounted circular cylinder has been considered by Porter and Newman [1–3], using an annular bed with a variable depth to refract the waves around the cylinder. Their computations show that near-zero values of the scattered energy can be achieved by optimizing the bathymetry of the bed. However the use of variable bathymetry may be impractical, especially in deep water. Thus the present work considers the possibility of cloaking a circular cylinder which is fixed on the free surface in a fluid of infinite depth, by surrounding it with one or more outer bodies. The dimensions of the inner cylinder are fixed, and the scattered energy is minimized at a value of the frequency where the product of the wavenumber and the cylinder draft is equal to one. Linearized time-harmonic motion of an ideal fluid is assumed.

Two specific types of surrounding structures are used to cloak the inner cylinder. The first is an array of outer cylinders which surround the inner cylinder, as shown in Fig. 1. This configuration was suggested by the work of Farhat et al. [4], who showed that a large number of small circular cylinders could be used to cloak an inner cylinder in problems governed by the two-dimensional wave equation. The second type is a continuous 'ring', such as a torus with constant cross-section or a non-axisymmetric body with varying cross-section. This type was suggested by the results for the arrays of cylinders, where the scattered energy is reduced progressively by increasing the number of cylinders and decreasing

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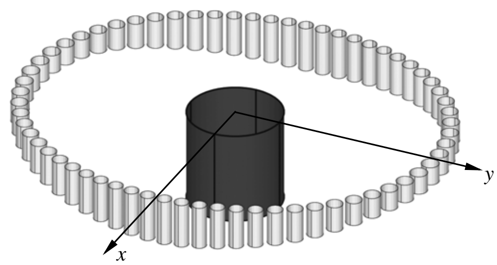

Fig. 1. Perspective view of the structure with $M = 64$ outer cylinders and $N = 15$ optimization parameters. Only the submerged surfaces are shown, with the upper edge of each cylinder in the plane $z = 0$.

their spacing. For both types it is shown that the scattered energy can be reduced to very small values by optimizing the dimensions and shape of the surrounding structure.

The structures are assumed to be symmetric about the planes $x = 0$ and $y = 0$, where the x-axis is in the direction of incident-wave propagation. Symmetry about $x = 0$ is suggested by reversing time (or conjugating the solution of the boundary-value problem with complex time-dependence). Thus, for any structure with no scattered waves, there also are no scattered waves if the incident-wave direction is reversed. This implies that the structure itself should be symmetric about $x = 0$. Symmetry about $y = 0$ is more obvious, since the incident-wave field is independent of y.

Preliminary results, which are more limited and less accurate, have been presented in [5]. The possibility of perfect cloaking with an axisymmetric structure was considered there. The results in [5] suggest that this might be possible, although it would be remarkable if perfect cloaking could be achieved with such a structure. The results presented here, which are more accurate, suggest that perfect cloaking can only be achieved with non-axisymmetric structures.

The theory and computational method are described in Sections 2 and 3. Results for the two types of surrounding structures are presented in Sections 4 and 5. These results are compared and discussed in Section 6.

2. Theory

A fixed structure consisting of two or more rigid bodies is situated on the free surface of the fluid, which is inviscid, incompressible, and extends to infinity in all horizontal directions. The fluid depth is infinite. Cartesian coordinates $\mathbf{x} = (x, y, z)$ are used with $z = 0$ the plane of the undisturbed free surface and z positive upward. Harmonic time-dependence is assumed, with the velocity potential

$$\Phi(\mathbf{x}, t) = \mathrm{Re}\left\{\phi(\mathbf{x})e^{i\omega t}\right\}. \tag{1}$$

Here t represents time, ω is the radian frequency, and ϕ is complex. The potential is a solution of the Laplace equation

$$\nabla^2\phi = 0 \tag{2}$$

in the fluid domain. Small amplitude motions are assumed, justifying the linearized free-surface boundary condition

$$K\phi - \phi_z = 0 \quad \text{on } z = 0, \tag{3}$$

where $K = \omega^2/g$ is the wavenumber and g is the gravitational acceleration. Subscripted lower-case letters denote partial differentiation. Since the fluid velocity vanishes at large depths,

$$\nabla\phi \to 0 \quad \text{as } z \to -\infty. \tag{4}$$

In the diffraction problem the structure is fixed, with plane progressive waves of amplitude A incident upon it. The Neumann boundary condition

$$\phi_n = 0 \tag{5}$$

is applied on the submerged surface S of the structure. The subscript n denotes the normal derivative, with $\mathbf{n}$ positive in the direction out of the fluid domain. The potential is defined in the form

$$\phi = A(\phi_I + \phi_S) \tag{6}$$

where ϕ_I is the incident-wave potential and ϕ_S is the scattering potential, both for unit amplitude A. Without loss of generality it can be assumed that the incident waves propagate in the positive x direction, and thus

$$\phi_I = \frac{g}{\omega}\,e^{Kz - iKx}. \tag{7}$$

The boundary-value problem is completed by imposing the radiation condition in the far-field, which can be expressed in the form

$$\phi_S \simeq \frac{g}{\omega}\frac{H(\theta)}{\sqrt{2\pi KR}}\,e^{Kz - iKR - i\pi/4} \quad \text{as } R \to \infty. \tag{8}$$

Here (R, θ) are polar coordinates with $x + iy = Re^{i\theta}$. The function $H(\theta)$, which represents the amplitude of the scattered waves, is known as the Kochin function. Following the analysis in [6], the Kochin function can be evaluated by applying Green's theorem, with the result

$$H(\theta) = \frac{\omega K}{g}\iint_S\left(\phi_{Sn} - \phi_S\frac{\partial}{\partial n}\right)e^{Kz + iK(x\cos\theta + y\sin\theta)}\,dS. \tag{9}$$

The normalized rate of scattered energy is given by the dual relations

$$E = \frac{1}{2\pi}\int_0^{2\pi}|H(\theta)|^2 d\theta = -2\mathrm{Im}\left\{H(0)\right\}. \tag{10}$$

The equivalence of these two relations follows from Green's theorem, as shown in [6], or more physically from the conservation of energy applied to the total potential (6). In other types of wave diffraction this equivalence is known as the optical theorem.

If the structure is symmetric about $x = 0$, the symmetric and anti-symmetric components of the potential ϕ_S satisfy Neumann boundary conditions on the body where the normal derivatives are real and imaginary, respectively. If there is no scattered energy these potentials vanish at infinity faster than a radiated wave, and satisfy the homogeneous boundary condition (3) on the free surface. It follows that the symmetric and anti-symmetric components of the potential are respectively real and imaginary throughout the fluid domain, assuming uniqueness. This property has important effects on the mean second-order pressure and drift force, as will be noted below.

3. Computational method

Our objective is to surround a prescribed inner body with one or more outer bodies which are optimized to minimize the scattered energy of the combined structure. The inner body is a circular cylinder with radius 0.5 m and draft 1.0 m. The optimization is performed at the wavenumber $K = 1$, using non-dimensional parameters normalized by the unit draft. The energy E is normalized by the corresponding value for the uncloaked cylinder, $E_0 = 0.0727344$. The energy ratio E/E_0 is defined in this manner.

The computational approach combines a three-dimensional radiation–diffraction code based on the boundary-integral-equation method (BIEM) with a multi-variate optimization code (PRAXIS).