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Waves propagating along a channel with ice cover

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Dedicated to the memory of Enok Palm in appreciation of his work and outstanding personality

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ABSTRACT

Linear progressive waves in a channel covered with ice sheet are studied. The channel is of rectangular cross section. The ice sheet is clamped to the walls of the channel. The thickness of the ice plate is constant. Deflections of the ice sheet are described by the linear elastic plate equation. The hydroelastic waves in the channel are combinations of waves propagating along the channel and sloshing waves. The problem is formulated with respect to the wave profiles across the channel. The problem is solved by the normal mode method for a channel of finite depth and by using the shallow water approximation for a channel of small depth. The dispersion relations of the hydroelastic waves and the characteristics of these waves are determined. It is shown that the shallow water approximation predicts well the dispersion relations for long waves. The dispersion relation for the wave, which does not oscillate across the channel, is well approximated by the corresponding dispersion relation of one-dimensional hydroelastic waves in an unbounded ice sheet. The wave profiles across the channel and the distributions of strains in the ice sheet are investigated. It is shown that the strains are maximum at the walls for long waves and at the centreline of the channel for short waves. The bending stresses across the channel are higher than the stresses along the channel for the conditions of the present study.

1. Introduction

Channels are important waterways in populated areas. They can be frozen in winter time. Safe transportation along frozen channels is an important problem. Channels can also be considered as a model of rivers in northern countries, where they are used for transportation in winter time giving access to remote areas. In the Northern Hemisphere in spring time, northern parts of such rivers can be covered with ice while their southern parts are already free of ice. This can be a reason for flooding on the boundary between the areas covered with ice and free parts of the river. To avoid flooding, the ice should be removed. This can be done by surface-effect-ships, for example [1,2]. The ship moves along a river at a certain speed generating deflection of the ice cover which is large enough to break the ice. If the dimensions of the ship are small compared with the width of the river, the problem of ice deflection can be solved by the approximation of ice sheet of infinite extent. This has been done in [1–6]. However, for narrow waterways the finite width of the ice sheet and conditions of its connection to the river banks can significantly affect the ice deflection and stresses in the ice cover.

The present study is motivated by three practical problems. First of them is the transmission of water waves propagating along a part of the river free of ice into the ice field. Is it possible to generate a wave that breaks the ice at large distance from its edge? The second problem is concerned with transportation along a frozen channel. In this problem, a vessel is represented by an external pressure spot, which moves along the channel. To solve this problem, we need to know dispersion relations, group velocities of the hydroelastic waves and the far-field behaviour of these waves. The third problem is concerned with a frozen channel downstream of a vibrating gate. The vibrations produce hydroelastic waves propagating along the channel. This is a classical wavemaker problem but for hydroelastic waves. The analysis presented in this paper is theoretical but its practical applications are clear. The problem under consideration can also be related to some industrial applications.

The dispersion relations between the length of a flexural wave and its frequency depend on the shape of the channel cross section. Below only rectangular cross sections are considered. However, the analysis can be generalised to any shapes of cross sections. For each calculated wave we find the maximum bending stress and identify the limiting wave amplitude and the places, where the bending stresses exceed the yield limit for the ice.

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The problem of linear waves propagating in a floating ice sheet received considerable attention. An excellent review of the subject has been given by Squire [7]. Different horizontal shapes of the sea ice were studied. The problem of a circular ice plate in waves was investigated for infinite (see Meylan and Squire [8]) and finite (see Peter, Meylan, Chung [9]) fluid depths. More general shapes of the ice sheet were studied by Meylan [10]. Evans and Porter [11,12] and Porter [13] studied the problem of cracks in the ice sheet. They determined the scattering properties of hydroelastic waves by an arbitrary number of infinite or finite straight-line cracks in an otherwise uniform plate floating on water of finite depth. Hydroelastic wave scattering by ice sheets of varying thickness were examined by Bennetts et al. [14].

The spectral theory for a two-dimensional elastic plate floating on water of finite depth was developed by Hazard and Meylan [15]. This theory describes the behaviour of a floating elastic plate in time-dependent linear waves. Hydroelastic waves in ice sheets frozen to vertical walls were less studied.

The linear two-dimensional and three-dimensional problems of hydroelastic waves reflected by structures with vertical walls were studied by Brocklehurst et al. [16,17]. The fluid was of finite depth. The ice sheet was of infinite extent and clamped to the vertical wall. The coupled problems of hydroelasticity were solved by using integral transforms. The ice deflections and strains in the ice sheets were analysed together with the vertical and horizontal forces acting on the rigid walls. Unsteady problems of the ice sheet clamped to a vertical wall can be studied by the spectral theory presented in [15].

The three-dimensional problem of a load moving along the ice sheet in the vicinity of a vertical wall was studied theoretically and numerically by Brocklehurst [18]. The ice sheet was frozen to the wall and was of infinite extent. The fluid was of finite depth. The study was concerned with whether the motion of the load can generate large enough sheet response to break the ice connection to the vertical wall. Both the distance of the load from the wall and the speed of the load varied in [18].

The linear analysis of the effect of an elastic ice cover on the wave propagation along a channel was published by Daly [19] and further refined by Steffler and Hicks [20]. Three linearised equations governing one-dimensional unsteady flow in rectangular ice-covered channels were used. These equations described the mass and momentum conservation of the flow along the channel and the elastic response of the ice cover. The ice cover connection to the channel banks was neglected. Daly [19] showed that transverse cracks could be caused by short hydroelastic waves (about 50 m) of very small amplitude (about 5 cm). Daly also determined the stress distribution in the ice cover for the entire spectrum of wave lengths.

Xia and Shen [21] presented a nonlinear dynamic analysis of the interaction between a water wave and a floating ice cover in river channels. They derived a one-dimensional weakly nonlinear fifth-order KdV equation for shallow water wave propagation in a uniform channel with a floating ice cover. It was shown that nonlinear cnoidal waves may also generate sufficient stresses to fracture the ice cover. Two-dimensional nonlinear hydroelastic waves were studied in [6,22].

Wave-generated fractures in ice covers were studied by Beltaos [23]. The problem of one-dimensional hydroelastic waves in a channel was solved with account for the presence of the ice edge or transverse cracks. The conditions of zero bending moment and shearing force were imposed at such edges. The ice cover was not clamped to the banks. It was concluded that the maximum stress for a wave propagating past an edge or crack into an undisturbed region is about the same as for the same infinitely long wave propagating under an edgeless cover.

Nzokou et al. [24] numerically studied the water waves resulting from a dam break for winter conditions, when a river is covered with ice. The dam-breaking waves in this study were strong enough to break the cover away from banks but not strong enough to create transverse cracks breaking the ice cover into many pieces. The derived model simulated the one-dimensional St. Venant equations coupled with the equation of a dynamic beam on an elastic foundation. The solution was obtained by the finite element method.

Fuamba et al. [25] studied dam-break wave propagation in a channel partially covered with ice between two consecutive dams. The developed numerical model included the ice cover which is clamped to the river banks and the upstream dam face. During the first stage the wave created by the dam break propagated towards the edge of the ice as a free-surface wave. Then the wave was divided into two: the pressure wave propagating under the ice cover and the free-surface wave propagating over the cover. Unsteady one-dimensional flow conditions through all three zones were simulated numerically. The dynamic response of the ice cover was described by the finite element method using the computed one-dimensional pressure distribution. It was reported that the model performed very well with a strong correlation between predicted and measured values.

The two-dimensional decoupled model of ice cover in a channel was presented by Nzokou et al. [26] for an infinitely long uniform channel. The finite element method was used to study the response of the ice sheet attached to the river banks to a one-dimensional water wave impinging on the domain. It was reported that there was a high stress concentration near the banks due to the fixed boundary conditions. However, close to the channel centreline, the stress distribution was found to be almost identical to that predicted by the one-dimensional analysis.

In the present paper we consider the coupled problem of linear hydroelastic waves propagating along a rectangular channel with ice cover clamped to the banks of the channel. The flow beneath the ice is governed by Laplace's equation. The deflection of the ice cover is described by the linear equation of elastic plate. The thickness of the plate is constant and negligible compared with the depth of the channel.

Formulation of the problem is given in Section 2. The formulae for the strains in the ice sheet and for maximum amplitudes of hydroelastic waves in a frozen channel are derived. The normal mode method [27,28] is applied to the problem in Section 3. The coupled problem of hydroelastic waves is reduced to a linear system of algebraic equations. The problem is also formulated within the shallow water approximation in Section 4. The numerical algorithms and the obtained results are presented in Section 5. Conclusions are drawn and future work is discussed in Section 6.

2. Formulation of the problem

We consider the wave of amplitude A and frequency ω propagating in the negative x-direction along a channel of depth H and width $2b$. The channel is of infinite extent in the x-direction (see Fig. 1). The channel is occupied with the liquid of density ρ_l. The liquid is inviscid and incompressible. The liquid is covered with an ice sheet of constant thickness h_i and rigidity $D = Eh_i^3/[12(1-\nu^2)]$, where E is Young's modulus of ice and ν is Poisson's ratio. The ice sheet is clamped to the walls of the channel.

The problem of waves in the channel covered with an ice sheet is formulated in non-dimensional variables within the linear theory of hydroelastic waves [1]. The half-width of the channel b is taken as the length scale, the ratio $1/\omega$, which is proportional to the wave period $T = 2\pi/\omega$, as the time scale, and the wave amplitude A as the displacement scale. The non-dimensional depth of the channel H/b is denoted by h. The irrotational flow of the liquid in the channel is described by the velocity potential $\varphi(x, y, z, t)$ with the scale $Ab\omega$. The deflection $w(x, y, t)$ of the ice sheet is described by the linear equation of elastic plate

$$\alpha\gamma w_{tt} + \beta\nabla^4 w = p(x, y, 0, t)$$
$$(-\infty < x < \infty, -1 < y < 1), \quad (1)$$