



Analytically approximate natural sloshing modes and frequencies in two-dimensional tanks



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ARTICLE INFO

Article history:

Available online 30 January 2014

This paper is dedicated to the memory of Enok Palm. He was for the first author the grand old man in theoretical fluid mechanics in Norway and made significant scientific contributions in many areas including interaction between free-surface waves, current and marine structures. His way of inspiring and showing interest in others work was special

Keywords:

Sloshing
Trefftz' solution
Natural modes and frequencies

ABSTRACT

Analytical approaches to nonlinear and linear sloshing problems need to know approximate natural sloshing modes expressed by continuously-differentiable harmonic functions. A new method for constructing those approximate modes as well as for a fast computing of the corresponding natural sloshing frequencies is proposed in the two-dimensional case. The method facilitates a parametric study of the natural sloshing frequencies in a prismatic tank associated with LNG (Liquefied Natural Gas) containers. The results are extensively compared with other approximate analytical solutions.

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1. Introduction

Finding the natural sloshing frequencies and modes is an important task in marine and spacecraft applications. Analytical solutions of the corresponding spectral boundary problem are a rare exception [1, ch. 4]; [2, ch. 1] and Finite Element and/or Boundary Element methods [1, ch. 10]; [3] are traditionally employed. Analytical approaches to linear and nonlinear liquid sloshing problems yield specific requirements to approximate natural sloshing modes. Even though the natural sloshing modes are formally defined in the mean liquid domain, the nonlinear multimodal method needs, for example, analytically-expressed continuously-differentiable *harmonic* natural sloshing modes which are continuously expandable above the mean liquid domain. The traditional numerical methods are then not applicable. A review on analytical approximate methods producing the analytically approximate natural sloshing modes is given by the authors in [1]. Interested

readers can also find extensive discussions on these methods in [4–6] (in the context of specific tank shapes).

The present paper constructs an approximate Trefftz solution of the two-dimensional natural sloshing problem. The solution appears as a linear combination of the so-called harmonic polynomials and their generalizations. The harmonic polynomials are a well-known instrument of different numerical methods, e.g., the Harmonic Polynomial Cell (HPC) method developed recently for surface wave problems [7]. The polynomials constitute a complete harmonic basis in the star-shaped domains [8–10]. The Trefftz solution is adopted by three alternative projective schemes which provide a fast computing of the corresponding natural sloshing frequencies.

Numerical experiments in [11, Sections 18, 26, and 27] with diverse harmonic functional base in projective [variational] schemes showed that a naive usage of these base in sloshing problems may not be efficient and/or accurate when the basic functions are not *a priori* constrained to a boundary condition and do not possess the corner-point asymptotic behavior at the contact line corner. We were familiar with this fact when working on [12] where approximate Trefftz-type natural sloshing modes were found for a two-dimensional circular tank. In that paper, we have constructed and employed two different harmonic functional sets, 'regular' and

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<http://dx.doi.org/10.1016/j.euromechflu.2014.01.005>

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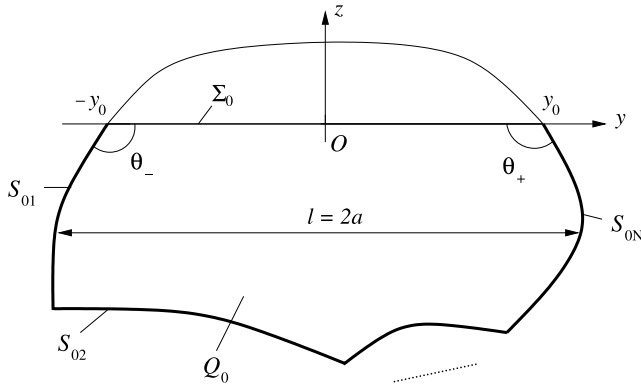


Fig. 1. The mean liquid domain Q_0 is bounded by the mean free surface Σ_0 and the mean wetted tank surface S_0 composed from N_s smooth curves S_{0i} , $i = 1, \dots, N_s$. The Oy axis is superposed with the interval $\Sigma_0 : \{z = 0, -y_0 < y < y_0\}$, so that the origin O is at the interval middle. The inner angles $0 < \theta_{\pm} < \pi$ at the corner points $(\pm y_0, 0)$ are formed by S_0 and Σ_0 .

'singular' at the contact corner points, so that they both *a priori* satisfy the zero-Neumann boundary condition on the mean wetted tank surface. The 'singular' functional set also possesses the corner-point asymptotics. A straightforward generalization of [12] can be based on the conformal mapping as proposed, e.g., in [13], where the Christoffel–Schwarz transformation is employed to approximate the mean liquid domain by a polygon. However, using the complex variables cannot provide, in general, an explicit harmonic expression of real variables for approximate natural sloshing modes. To get the needed analytically-expressed approximate natural sloshing modes, we have to follow an alternative way. Furthermore, we need in the future to generalize the method to the three-dimensional case. The present approach, generally, allows for that.

Section 2 formulates the problem. In Section 3, we construct two κ -families of harmonic functions in which the positive real number κ is associated with the spectral parameter proportional to the square of the natural sloshing frequency. The harmonic functions satisfy the spectral boundary [Robin] condition on the mean free surface. The first family is the harmonic polynomials but the second one possesses the asymptotic behavior of the eigenfunctions at the intersection points of the mean free surface and the wetted tank walls. A linear combination of the harmonic functions with unknown weight coefficients constitutes an approximate Trefftz solution of the original spectral boundary problem. To find κ and the weight coefficients, we employ three alternative projective schemes described in Sections 4.1–4.3, respectively. The schemes are efficient for computing the lower natural sloshing frequencies and modes. Their advantages and disadvantages are extensively discussed. In Section 4.4, the projective schemes are verified by comparison with analytical and numerical solutions.

The approximate Trefftz solution is employed in Section 5 for a parametric study of the lowest natural sloshing frequencies for a two-dimensional prismatic tank associated with LNG (Liquefied Natural Gas) containers. The Trefftz approximations of the nondimensional spectral parameter κ play the role of reference values which are compared with approximate κ following from other, simplified analytically approximate solutions and estimates. The latter includes the shallow water approximation, Faltinsen and Timokha formula [1, ch. 4], and novel approximate formulas firstly derived in the present paper. Based on the parametric study, we formulate a guidance for practically-oriented readers who want to estimate the lowest natural sloshing frequencies for a two-dimensional prismatic tank without involving a numerical solver of the original spectral boundary problem.

2. Statement of the problem

The task consists of constructing analytically approximate natural sloshing modes φ (defined within to a multiplier) and frequencies $\sigma = \sqrt{g\kappa}$ (g is the gravity acceleration) which are the eigensolution of the spectral boundary problem

$$\begin{aligned} \nabla^2 \varphi &= 0 \quad \text{in } Q_0; & \frac{\partial \varphi}{\partial n} &= 0 \quad \text{on } S_0; \\ \frac{\partial \varphi}{\partial n} &= \bar{\kappa} \varphi \quad \text{on } \Sigma_0; & \int_{\Sigma_0} \varphi dS &= 0. \end{aligned} \quad (1)$$

Here, Q_0 is the mean liquid domain, S_0 is the mean wetted tank surface, and Σ_0 is the mean liquid surface (see, Fig. 1). Henceforth, we assume that Q_0 is the star-shaped domain relative to the origin O (the star-shaped domain means that for any point q in Q_0 , the line segment joining O and q lies entirely within Q_0). The origin O is superposed with the middle of $\Sigma_0 : \{z = 0, -y_0 < y < y_0\}$. Furthermore, the mean wetted tank surface S_0 is constituted by the smooth finite length curves S_{0i} , $i = 1, \dots, N_s$. The inner angles $0 < \theta_{\pm} < \pi$ at the corner points $(\pm y_0, 0)$ are due to intersection of S_0 and Σ_0 . Interior corner points are possible.

The spectral boundary problem (1) will be considered in the *nondimensional statement* for which the characteristic spatial dimension is associated with the half of the maximum horizontal tank width, $a = \frac{1}{2}l$ (see, Fig. 1). This means that the *nondimensional spectral parameter* κ computes the circular natural sloshing frequencies by the formula

$$\sigma = \sqrt{g\kappa/a}. \quad (2)$$

We will construct the κ -families of harmonic functions which satisfy the Robin [spectral] condition on Σ_0 within a real positive number κ . The families are the harmonic polynomials and their 'singular'-type generalization possessing the corner-point asymptotics at $(\pm y_0, 0)$. A finite sum by these functions (within unknown weight coefficients) gives an approximate Trefftz solution of (1) providing the Laplace equation and the Robin condition on Σ_0 are *a priori* satisfied. To approximate the zero-Neumann condition on S_0 and, thereby, find the spectral parameter κ and the unknown weight coefficients in the Trefftz solution, we will adopt three alternative projective schemes.

3. Harmonic functional base and the approximate Trefftz solution

3.1. Harmonic polynomials as a 'regular' functional basis

The so-called harmonic polynomials are polynomials by spatial coordinates which satisfy the Laplace equation. They constitute a complete harmonic functional basis in the star-shaped domains (relative to the origin). The completeness theorems can be found in [8,9] (see, also references in [10]). The two-dimensional harmonic polynomials are

$$w_n^{(1)} = \rho^n \cos(n\theta) \quad \text{and} \quad w_n^{(2)} = \rho^n \sin(n\theta), \quad n = 0, 1, \dots, \quad (3)$$

rewritten from polar (ρ, θ) to the Cartesian coordinate system Oyz . The polynomials $w_n^{(1)}(y, z)$ and $w_n^{(2)}(y, z)$ can be derived via the recursive relations

$$\begin{aligned} w_0^{(1)} &= 1; & w_0^{(2)} &= 0; & w_n^{(1)} &= yw_{n-1}^{(1)} - zw_{n-1}^{(2)}; \\ w_n^{(2)} &= yw_{n-1}^{(2)} + zw_{n-1}^{(1)}, & n &\geq 1. \end{aligned} \quad (4)$$

The first-order spatial derivatives of $w_n^{(1)}$ and $w_n^{(2)}$ read as

$$\begin{aligned} \frac{\partial w_n^{(1)}}{\partial y} &= nw_{n-1}^{(1)}; & \frac{\partial w_n^{(2)}}{\partial y} &= nw_{n-1}^{(2)}; \\ \frac{\partial w_n^{(1)}}{\partial z} &= -nw_{n-1}^{(2)}; & \frac{\partial w_n^{(2)}}{\partial z} &= nw_{n-1}^{(1)}, \quad n \geq 1. \end{aligned} \quad (5)$$

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