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Incompressible impulsive wall impact of liquid bodies

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ABSTRACT

Analytical leading-order solutions are given for various liquid bodies in translational motion that hit a plane wall in impulsive impact at constant velocity. The initial velocity field and the associated virtual masses are calculated for selected liquid bodies of uniform density. We consider 2D and 3D wedges, a cone, the semi-circle and the hemisphere. Sideways force impulses are obtained for 2D bodies. The loss of mechanical energy during impact is calculated for a 2D wedge.

1. Introduction

The present paper is concerned with incompressible impulsive impact of liquid bodies on plane walls. The theory of finite fluid masses impacting on solid walls was initiated and later followed up by Cooker and Peregrine [1–3]. Incompressible liquid impact has technological importance for breaking waves that suddenly hit coastal and marine structures.

In analogy with impulsive sloshing in open containers [4], there are four basic time scales for liquid impact on rigid walls: (i) The acoustic time scale. (ii) The incompressible impulsive time scale. (iii) The gravitational time scale. (iv) The viscous time scale.

The acoustic time scale for liquid impact can usually be disregarded because it is of the order of milliseconds, and the resulting displacements in the liquid will be negligible. The incompressible impulsive flow lacks an explicit time scale, so it sets in immediately after the acoustic stage is finished. Gravitational and viscous effects after impact need time to develop, and these will not influence the initial impulsive flow.

The topic of incompressible impulsive impact of fluid bodies on plane walls was already introduced by Milne-Thomson [5]. The exact solution for a cone was given in this classic textbook, but it was presented in a modest way as an exercise for the reader. The even simpler solution for a 2D wedge was not mentioned in the book, but it has been explored by Cooker [6].

We take an academic approach to the topic of incompressible impulsive impact. The idealization of a flat impact means that there is a finite area of instantaneous collision between the fluid body and the solid wall. Our idealized approach complements the existing research of liquid impact, which has had an applied technological scope since the first paper by Cooker and Peregrine [1]. The focus has been on the understanding of extreme loads on harbors and marine constructions due to impacting seawater. These liquid masses have their origin in breaking ocean waves. Incompressible liquid impact on rigid walls has not yet become established as an academic topic in its own right. The present work is a modest attempt to improve this situation. In establishing a topic academically, one should explore systematically the simplest nontrivial cases, and develop analytical theories from first principles. Moreover, one should provide identity to this particular scientific topic by clarifying its relations to the neighboring branches of science.

Incompressible impact of finite fluid bodies on plane rigid walls is linked to three fields of engineering science:

(1) The broad field of slamming, which is concerned with solid bodies colliding with fluid masses. This is a well-established field of research, see the review articles by Korobkin and Pukhnachov [7] and Faltinsen, Landrini and Greco [8]. The latter authors identified liquid impact as a subfield of slamming, and followed it up in a separate paper [9]. Our work is restricted to the idealized case of fluid bodies that are at rest before being hit by a plane wall in uniform motion.

(2) Impulsive free-surface flows, driven by moving walls or objects that are located inside the fluid. The topic most relevant here is impulsive sloshing [4], where a container with fluid in hydrostatic equilibrium is suddenly put into motion. A generalization could merge impulsive sloshing with the present topic of impacting liquid bodies. This is achieved if we abandon the restrictions of having either a flat wall of impact or an initially flat liquid surface. Instead we assume two types of curved boundaries that confine the

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total fluid mass: (i) Parts of the boundary is a curved surface that is free to move. (ii) The remaining boundary is a curved solid wall being forced impulsively into motion.

(3) Dam breaking, or more generally gravitational collapse, where liquid bodies resting on a horizontal plane are instantaneously released to flow freely under gravity. This topic was studied by Penney and Thornhill [10], with accompanying experiments reported by Martin and Moyce [11]. These problems relate to fluid impact problems where the relative motion between the liquid masses and the planes of impact take place with constant acceleration instead of constant velocity. In the present work we will demonstrate a mathematical analogy between dam breaking and liquid impact on plane walls, but its validity is limited to the initial flow only.

The potential flow theory of incompressible dam breaking is troublesome because of the mathematical singularity that appears at the waterline where a vertical liquid face meets a horizontal wall. The traditional shallow-water theory of dam breaking [12] tries to avoid the singularity by taking the inaccurate assumption of hydrostatic pressure. Stansby et al. [13] simulated the full potential theory of wave breaking, but had to invent numerical artifacts in order to avoid the waterline singularity. The only consistent way of dealing with the contact line singularity is to take the singular flow field as the outer field in a matched-asymptotics sense, and develop inner solutions to resolve the singularity [14].

The present paper is restricted to the development of leading-order outer solutions for a variety of liquid body geometries. We do not solve any gravitational collapse problem, but a closely related mathematical problem: incompressible impulsive impact of liquid bodies on flat walls. In the present work we only consider the initial impact flow, studying liquid bodies for which an exact analytical solution can be found. 3D cylinders are excluded from the present work, and will be analyzed in a second paper.

The incompressible impulsive impact on flat walls is a special case of the more general pressure–impulse theory of liquid impact [1–3]. The type of impact that we consider lets a finite flat portion of the liquid surface hit the flat wall head on.

As pointed out by Greco, Landrini and Faltinsen [9], the theory of impacting liquid bodies may apply to green-water phenomena of seawater on ship decks. Nielsen and Mayer [15] considered mild green-water flows with a gradual start, where the full Navier–Stokes equations are appropriate. The present work applies only to instantaneous flows due to sudden impact, where gravity and viscosity will be negligible.

2. Model assumptions and formulation

A finite liquid body in 2D or 3D is considered, being initially at rest. At time $t = 0$ this inviscid and incompressible fluid body is hit by a plane wall $z = 0$ that is in uniform motion with velocity W in the z direction. Lord Kelvin's circulation theorem implies that the induced inviscid flow is irrotational. The constant density of the liquid body is ρ. The assumption of incompressible flow implies $\nabla \cdot \vec{v} = 0$ leading to Laplace's equation

$$\nabla^2 \phi = 0 \tag{1}$$

for the velocity potential ϕ, where the velocity field is $\vec{v} = \nabla \phi$.

The horizontal wall $z = Z(t)$ moves vertically and hits the flat front of the stagnant fluid body at $t = 0$, inducing immediate incompressible flow in the fluid body. The position of the moving wall is

$$Z(t) = H(t)(Z_1 t + Z_2 t^2 + Z_3 t^3 + \cdots), \tag{2}$$

where $H(t)$ is the Heaviside unit step function. The contact between the fluid body and the moving wall starts instantaneously at $t = 0^+$, with a finite initial contact area. The flow due to the moving wall with position $z = Z(t)$ may be expressed by the small-time expansions of the potential and the pressure

$$\begin{aligned} \phi(x, y, z, t) &= H(t)(\phi_0(x, y, z) + t\phi_1(x, y, z) \\ &\quad + t^2 \phi_2(x, y, z) + \cdots), \end{aligned} \tag{3}$$

$$\begin{aligned} p(x, y, z, t) &= p_{-1}(x, y, z)\delta(t) + H(t)(p_0(x, y, z) \\ &\quad + t p_1(x, y, z) + t^2 p_2(x, y, z) + \cdots). \end{aligned} \tag{4}$$

$\delta(t)$ denotes Dirac's delta function. Possible waterline singularities may limit the validity of these Taylor series to one term only.

The Bernoulli equation is given by

$$\frac{\partial \phi}{\partial t} + \frac{p}{\rho} + \frac{|\vec{v}|^2}{2} + gz = 0. \tag{5}$$

Here we have included a gravitational acceleration g in the $-z$ direction, even though the leading-order impulsive flow is without gravity. The fluid body is surrounded by vanishing density. The kinematic condition at the bottom plane in forced motion is

$$\frac{\partial \phi}{\partial z} = \frac{dZ}{dt}, \qquad z = Z(t). \tag{6}$$

The flow is impulsively started during $0 < t < 0^+$. Integrating Bernoulli's equation (5) over this infinitesimal time interval gives

$$\phi = 0, \qquad t = 0^+, \quad \text{along the free contour of the fluid body.} \tag{7}$$

We will solve the leading-order flow problem at $t = 0^+$. Then we introduce $W = Z_1$ and put all $Z_n = 0$ for $n > 1$. We will solve a mixed boundary value problem with Laplace's equation for ϕ_0

$$\nabla^2 \phi_0 = 0, \tag{8}$$

applying an inhomogeneous Neumann condition at the bottom

$$\frac{\partial \phi_0}{\partial z} = Z_1 = W, \qquad z = 0, \tag{9}$$

and the homogeneous Dirichlet condition $\phi_0 = 0$ along the open contour. Alternatively we may write $\phi_0 = Wz + \phi_0'$ and solve a mixed problem for ϕ_0'.

The potential ϕ' represents the flow immediately after impact in a (primed) coordinate system (x', y', z') defined by

$$(x', y', z') = (x, y, z - Wt), \tag{10}$$

implying $\phi' = \phi - Wz$. In the primed reference system, the liquid body impacts at the static wall $z' = 0$. Before the impact at $t = 0$, the liquid body moves with a uniform translational velocity W in the $-z'$ direction.

In the original (unprimed) coordinate system the liquid body at rest is hit by the wall that moves in the z direction with velocity W. The impulsive pressure is given by $p_{-1} = -\rho\phi_0$, which gives the force impulse

$$F_{-1z} = -\rho \int_{A_z} \phi_0(x, y, 0) dx dy = m_z W. \tag{11}$$

A_z is the set of points on the wall that is initially wetted by the impact. We define the virtual mass m_z for the impulsive motion of the fluid body forced by the impacting wall. The total fluid mass is M.

For a general 3D body, there are four lateral flow components, in the $+x$, $-x$, $+y$ and $-y$ directions, all of these perpendicular to the plane of impact $z = 0$. We denote the virtual masses for the lateral flow in these directions by m_x, m_{-x}, m_y, m_{-y}, respectively. All of these virtual masses are defined by an appropriate effective fluid momentum divided by W. There is no lateral momentum before the wall impact. Therefore it follows from Newton's third law and the conservation of momentum that $m_x = m_{-x}$ and $m_y = m_{-y}$.