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Bottom pressure distribution under a solitonic wave reflecting on a vertical wall

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HIGHLIGHTS

- Travelling and fully reflected solitonic waves are propagated numerically.
- A first approach solves numerically the fully nonlinear potential equations.
- A second one is based on the solution of the Green–Naghdi system of equations.
- Bottom pressure distributions and runup heights obtained are analysed and compared.
- Green–Naghdi equations predict satisfactorily the evolution of the bottom pressure.

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ABSTRACT

The bottom pressure distribution under solitonic waves, travelling or fully reflected at a wall is analysed here. Results given by two kind of numerical models are compared. One of the models is based on the Green–Naghdi equations, while the other one is based on the fully nonlinear potential equations. The two models differ through the way in which wave dispersion is taken into account. This approach allows us to emphasize the influence of dispersion, in the case of travelling or fully reflected waves. The Green–Naghdi model is found to predict well the bottom pressure distribution, even when the quantitative representation of the runup height is not satisfactorily described.

1. Introduction

The description of the bottom pressure distribution beneath nonlinear surface waves has several motivations. Among these motivations, bottom pressure sensors have long been used to measure surface waves, mainly in the low-frequency range. In the case of very long waves, like tides and tsunamis, the pressure is hydrostatic, and recovering of surface elevation from the data of bottom sensors is quite straightforward. Wind waves, however, are not long even in the coastal zone, and non-hydrostatic (dispersive) corrections play a significant role. Spectral methods, based on transfer functions, are often used to reconstruct the water elevation taking into account the assumption of linearity of waves [1–8]. Meanwhile, the linear hypothesis does not hold when the amplitude of the waves increases, and as it is shown in [3], the linear prediction for largest waves underestimates the results of about 15%. The pressure under nonlinear progressive periodic and solitary waves is found in [9–11], and a map from pressure to surface is presented in [12–14].

The wave behaviour near the coast (cliffs or vertical barriers) in the process of the wave reflection is more complicated. For instance, the relation between wave elevation and bottom pressure is not straightforward, due to the interaction of the incident and

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reflected waves [15]. Furthermore, the head-on-head collision of solitary waves or the soliton reflection from the wall is not elastic, and a dispersive tail appears behind the soliton [16–22]. If the solitary wave has large amplitude, near the wall the wave amplitude in the resulting field may exceed the limiting value and the instability on the wave crest induces the highest slash on such walls [23]. In such conditions, a thin jet generated on the crest of colliding waves was observed numerically in [24], where a detailed study of the collision of high amplitude solitary waves is provided. This jet on the crest of colliding waves should lead to the significant decreasing of the pressure and therefore in the wave action on the wall, but the pressure in such nonlinear wave field, in our knowledge, has not been studied yet.

The question of modelling bottom pressure distribution under nonlinear water waves is often important in problems involving wave propagation, transformation, and beach runup in fairly large and complex areas. The computational cost inherent to such problems being extremely important, the knowledge of simplified models is essential for simulation purposes. Indeed, since the Indian Ocean tsunami of 2004, it became obvious that nonlinear and dispersive effects could play a major role in such processes. Recent progress in Boussinesq-Type models allowed them to describe these effects better. This is why several of these models (FUNWAVE, COULWAVE, GloBouss) nonlinear and dispersive are often used to model tsunami waves. These tsunamis might be of seismic origin [25], generated by submarine landslides [26], due to volcanic eruptions [27], or even storm induced [28].

Still, the limitations of these methods are known. The classical equations (see, for example, [29]) incorporate only weak dispersion and weak nonlinearity, and in practice their range of applicability is limited to $kh < 0.75$. This shortcoming has attracted considerable attention in the recent past, period during which a number of enhanced and higher-order Boussinesq equations have been formulated to improve both linear and nonlinear properties. Unfortunately, many of these models are numerically unstable when dealing with large waves [30]. Thus, Boussinesq-type models of various levels of nonlinearity and dispersion are studied theoretically, using and validating several theories [31].

The Green–Naghdi model was the first model taking the full nonlinearity into account, in the framework of weak dispersion. This model is of particular concern when studying Boussinesq-type models, due to its specific mathematical properties. Indeed, as it is pointed out by Le Metayer et al. [32], the mathematical founding principles of this system are rather strong. The derivation of the Green–Naghdi model was achieved through a variational formulation of the Euler equations by Miles and Slmon [33]. A mathematical justification of the Green–Naghdi model was performed by Makarenko [34], and Alvarez-Samaniego and Lannes [35]. Camassa et al. [36] proposed a Hamiltonian formulation of the Green–Naghdi model.

From a physical point of view, the Green–Naghdi system of equations was studied in various contexts, and it was shown to predict important features of the flow (excluding wave breaking) accurately over a wide range of parameters (see for instance [37]). This system of equations is also used to describe two-layer flows. In this framework, it is named the Choi–Camassa system.

The main goal of the given paper is to analyse the ability of the Green–Naghdi model to describe bottom pressure variation in the process of the soliton reflection from a vertical wall. This analysis is performed numerically in the framework of fully nonlinear Euler equations (Section 2) and the weakly dispersive fully nonlinear Green–Naghdi system (Section 3). Results of computations for travelling and reflected waves are discussed respectively in Sections 4.1 and 4.2.

2. Numerical solution of the fully nonlinear equations

2.1. Basic equations of the problem

The problem is solved by assuming that the fluid is inviscid, incompressible, and the motion irrotational. Thus, the velocity field is given by $u = \nabla\phi$, where the velocity potential $\phi(x, z, t)$ satisfies Laplace's equation. The domain is bounded by the free surface, a horizontal solid bottom and two vertical solid walls. The horizontal and vertical coordinates are x and z respectively whereas t is the time. The still-water level lies at $z = 0$, and the horizontal impermeable bed is located at $z = -H$. The dynamic free surface condition states that the pressure at the surface, $z = \eta(x, t)$, is nil. Assuming the free surface to be impermeable, the problem to be solved is Laplace's equation with the kinematic and dynamic free surface boundary conditions, and the bottom boundary condition.

$$\begin{cases} \Delta\phi = 0 & \text{in } -h \le z \le \eta(x,t), \\ \dfrac{\partial\eta}{\partial t} + \dfrac{\partial\phi}{\partial x}\dfrac{\partial\eta}{\partial x} = \dfrac{\partial\phi}{\partial z} & \text{on } z = \eta(x,t), \\ \dfrac{\partial\phi}{\partial t} + \dfrac{(\nabla\phi)^2}{2} + g\eta = 0 & \text{on } z = \eta(x,t), \\ \dfrac{\partial\phi}{\partial z} = 0 & \text{on } z = -H \end{cases} \quad (1)$$

g being the acceleration due to gravity and ρ the water density. Once the velocity potential and its gradient are known in the fluid, the bottom pressure is obtained by using Bernoulli's equation

$$\frac{p}{\rho} = gh - \frac{\partial\phi}{\partial t} - \frac{(\nabla\phi)^2}{2} \quad \text{on } z = -H. \quad (2)$$

2.2. Numerical approach

A Boundary Integral Equation Method (BIEM) is used to solve the system of equations (1) with a mixed Euler Lagrange (MEL) time marching scheme. Full details of this numerical approach can be found in [38]. This method was already used to investigate the propagation of solitonic waves in [24].

The method is based on the use of Green's second identity, to solve Laplace's equation for the velocity potential.

$$\int_{\partial\Omega} \Phi(\mathrm{P})\frac{\partial G}{\partial n}(\mathrm{P},\mathrm{Q})d\ell - \int_{\partial\Omega} \frac{\partial\Phi}{\partial n}(\mathrm{P})G(\mathrm{P},\mathrm{Q})d\ell = c(\mathrm{Q})\Phi(\mathrm{Q}), \quad (3)$$

where G is the free space Green's function. The fluid domain boundary $\partial\Omega$ is $\partial\Omega_B \cup \partial\Omega_F$, which correspond respectively to solid boundaries and to the free surface boundary. Since P and Q refer to two points of the fluid domain, and since $c(\mathrm{Q})$ is given by

$$c(\mathrm{Q}) = \begin{cases} 0 & \text{if Q is outside the fluid domain}\,\Omega \\ \alpha & \text{if Q is on the fluid boundary } \partial\Omega \\ 2\pi & \text{if Q is inside the fluid}\,\Omega, \end{cases} \quad (4)$$

a discretization of this integral equation can be obtained. Time stepping is performed by means of a fourth order Runge & Kutta scheme, with a constant time step. The bottom pressure is calculated by using a finite-difference method.

2.3. Initial condition

We consider a rectangular wave tank of length L and constant depth h with two vertical solid walls located at its ends. The horizontal length of the domain, L, is assumed to be large enough to avoid any perturbation generated from the vertical walls during the computational time of the simulations. For the results concerning propagative waves, a single solitary wave is considered, initially