European Journal of Mechanics B/Fluids 48 (2014) 165–173

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Using vortex strength wake oscillator in modelling of vortex induced vibrations in two degrees of freedom

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ARTICLE INFO

Article history:
Received 24 July 2013
Received in revised form
18 December 2013
Accepted 9 May 2014
Available online 14 June 2014

Keywords:
Vortex-induced vibration
Two degrees of freedom
Wake oscillator model
Wake strength
Fluid–structure interaction

ABSTRACT

A new wake oscillator model is established to predict the structural response characteristics of vortex induced vibration (VIV) in two degrees of freedom. Based on the two-dimensional potential flow approach, the streamwise and transverse fluctuating fluid forces acting on structure are simplified and quantified. The work–energy balanced between the fluid and the structure leads to establish the coupled dynamic model by introducing a displacement variable related to the strength of nascent vortex. Analysis and prediction of the amplitudes, frequencies and phase angles of x–y motions of a rigid 2-D circular cylinder, along with a comparison to the existing experimental results, show that the numerical solutions of the present model can qualitatively and quantitatively capture the important features of VIV. Moreover, the x–y trajectory displays a crescent shape at cross-flow approaching peak amplitude. The reduced-order model also used in predicting VIV response of a 3-D top tensioned riser undergoing a stepped current is presented. The simulation results highlight the combination of standing wave and traveling wave occurring in structure vibration. The trajectories exhibit various patterns of figure-of-eight orbital motions and phase angles between the in-line and cross-flow motion change more rapidly in the standing wave region.

1. Introduction

Vortex-induced vibration (VIV) is a typical flow–structure interaction phenomenon and can cause problems in many areas related to the significant structural failure from fatigue. In the recent development of offshore industry, VIV must be taken into account in engineering design, such as drilling risers, free spanning pipelines, and marine cables. Some systematic and comprehensive reviews attempted to introduce and summarize the physical phenomena, existing experiments and theoretical analyses of VIV, can be found in [1–5].

For predicting the dynamic response of structure experiencing vortex induced vibration, the computational fluid dynamics (CFD) techniques are widely adopted to compute the fluid forces on the rigid body by calculating the flow field information. The CFD approaches, including the direct numerical simulations (DNS), Reynolds Navier–Stokes equations (RANS), large eddy simulations (LES) and vortex element methods (VEM) mostly consist of solving the time dependent of Navier–Stokes equation directly or approximately, however, are limited by heavy computational requirements, which is very difficult to satisfy the practical engineering requirements up to now.

Apart from numerical simulations, semi-empirical models as an alternative approach, mainly including wake-oscillator models, single degree-of-freedom (SDOF) models, force-decomposition models and variational approaches, spring up for predicting VIV responses due to their wide industrial applications. The main feature of phenomenological approach is that the dynamic behavior of vortex shedding is modeled by using a dynamic system. A detailed review on VIV modeling has been given by Gabbai and Benaroya [2].

In particular, the wake-oscillator coupled models have been greatly improved in recent years. A wake oscillator used to model the dynamical behavior of the cylinder's wake, was noted and introduced by Birkhoff and Zarantanello [6] and Bishop and Hassan [7]. Further, a specific math expression of wake oscillator was firstly given by using nonlinear oscillator equation, namely Rayleigh or Van der Pol equation [8–10]. There is a considerable number of wake-oscillator models discussed in the literatures. In a recent paper, Facchinetti et al. [11] presented an important insight on the dynamics of wake oscillator models for single degree of freedom VIV, considering three different types of coupling effects of the cylinder motion on the wake oscillator. An acceleration coupling can be successfully used for reproducing the general mechanisms in VIV, such as lock-in domains, hysteresis and Griffin plots. Ogink and Metrikine [12] tried to improve the original model of Facchinetti et al. to predict both the free and forced vibration by introducing frequency dependent couplings.

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http://dx.doi.org/10.1016/j.euromechflu.2014.05.002
0997-7546/

In most previous work, even though a considerable number of papers only focused on the problem of a transversely vibrating circular cylinder, the case of VIV in two degrees of freedom is more common and practical. Recently, it has become clear that the response in the in-line (IL) direction, though not as large as cross-flow (CF) direction, is significant for long slender cylinders. Baarholm et al. [13] revealed that the maximum fatigue damage on the circumference of the riser cross section was in general close, either the IL or the CF fatigue damage contribution. Therefore, the fatigue damage arising from IL vibration cannot be neglected. Further, it is quite necessary to present a wake-oscillator coupled model for predicting the two degrees of freedom VIV. Furnes et al. [14], Ge et al. [15] and Li Xiao-min et al. [16] respectively presented a coupled model with a pair of van der Pol oscillators, each for one direction, and Srinil and Zanganeh [17] presented double Duffing–van der Pol (structural-wake) oscillators with two structural equations, to describe the in-line/cross flow VIV. Nevertheless, it is inevitable that more empirical parameters in the model need to be determined. Furthermore, the relationship between two nonlinear wake oscillators is difficult to be confirmed as a result of the same fluctuating mechanisms of vortex shedding.

In this paper, a modification to the previous wake oscillator model for predicting two degrees of freedom VIV is performed based on a new independent variable for the wake oscillator. The paper is structured as follows. In Section 2, a reduced-order model is established to model 2-D VIV of a circular cylinder. Two-dimensional potential flow method is used to estimate the fluctuating forces on the structure induced by vortices in the wake. Thus, the in-line and cross-flow fluid forces can be quantified through the vortex strength governed by a Rayleigh oscillator based on the energy transfer employing the vortex force component from the fluid to the structural motion. The model is solved by numerical method and compared in detail with rigid cylinder experiments. In Section 3, the present model is extended to 3-D VIV of a top tensioned riser. The analysis and prediction of in-line and cross-flow VIV response of long slender cylinder are carried out and display good comparisons with experimental results in the literatures. In Section 4, the conclusions of this paper are given.

2. Two-dimensional model of rigid circular cylinder

2.1. Structural governing equation

A reduced-order mathematical model simulating the two degrees of freedom of an elastically supported rigid circular cylinder with diameter D under steady uniform fluid with an upstream velocity U is developed. A Cartesian coordinate system is defined to describe the model in the XOY plane, Fig. 1. In the case of two degrees of freedom, a system is concerned with precisely the same mass and spring stiffness in the X- and Y-direction. Therefore the motion equation of cylinder can be extended to two degrees of freedom form as,

$$(m_s + m_a)\ddot{\mathbf{r}} + \left(c_s + c_f\right)\dot{\mathbf{r}} + k\mathbf{r} = \mathbf{f}_V \quad (1)$$

where overdots denote derivatives with respect to the dimensional time t; total mass $m = (m_s + m_a)$, m_s is the cylinder mass, m_a is the fluid added mass, which represents the inviscid inertial effects, namely $m_a = C_a \pi D^2 \rho/4$ and C_a is the added mass coefficient; c_s is the linear structure damping; c_f is the fluid-added damping [18], defined as $c_f = \gamma \omega_{st} \rho D^2$, where γ is the stall parameter which has been discussed in detail [19,20], and ω_{st} is the vortex-shedding angular frequency. For the sake of simplicity, this term in this paper is dealt with the linear form presented by Facchinetti et al. [11]; k is the linear spring stiffness; $\mathbf{r}$ is the displacement vector of rigid body, which is defined as $\mathbf{r} = X + \mathrm{i}Y$ or a point (X, Y) in complex field; $\mathbf{f}_V$ represents the time-varying fluid forces as a result of vortex shedding in the cylinder wake acting on the cylinder, which can be expressed by a complex representation as $\mathbf{f}_V = F_X + \mathrm{i}F_Y$. Further, Jauvtis and Williamson [21] put forward a good approximation to the in-line and cross-flow displacements provided by the equations as,

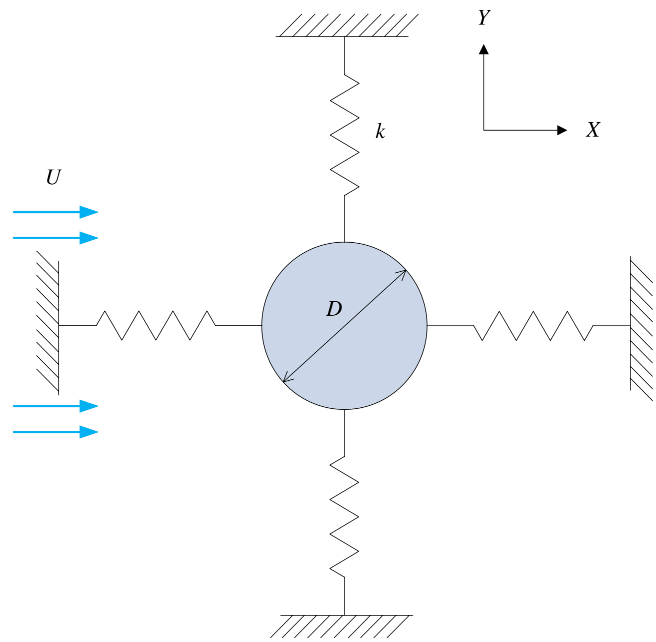

Fig. 1. A schematic model of vortex-induced vibration of 2-D freely oscillating rigid cylinder.

$$X(t) = A_X \sin(2\omega t + \theta), \qquad Y(t) = A_Y \sin(\omega t) \quad (2)$$

where ω is the oscillation angular frequency; θ is the phase angle, which can be used to describe the relationship between X and Y motions. For clarity, the schematic of the X–Y trajectory shapes for various values of phase angle is plotted in Fig. 2.

2.2. Fluctuating fluid forces

For a certain range of Reynolds number, when the uniform fluid flows past a stationary circular cylinder, the boundary layer may be separated from both sides of the object and vortex street may be formed from the alternating periodic shedding of vortices. The wake pattern of periodical shedding vortices behind the stationary cylinder can be simply modeled by a staggered double rows point vortices with equal magnitude Γ and opposite sign of vortex strength, Fig. 3. It is further assumed that the whole flow field remains inviscid, incompressible and outside the vortices, irrotational. The viscous diffusion of vortices and the stability of the vortex street are not taken into consideration. The entire flow field can be divided into two parts, the near-wall control zone and the wake zone. The near-wall control zone is a thin region enclosing the body surface, in which the near-wall nascent vortex is generated. The wake zone is an infinite region outside the surface of near-wall control zone to infinite space, including a specified number of isolated vortices with constant strength and a shedding vortex from the near-wall control zone, Fig. 3.

By using circle theorem, the complex velocity potential w can be described in the presence of n isolated vortices and m nascent vortices in the feeding layers and wake. Therefore, the representation of the flow field can be simplified as,

$$w = U\left[\mathbf{r} + \frac{1}{4}\frac{D^2}{\mathbf{r}}\right] + \frac{\mathrm{i}}{2\pi}\sum_{k=1}^{N}\Gamma_k \ln(\mathbf{r} - \mathbf{r}_k) - \frac{\mathrm{i}}{2\pi}\sum_{k=1}^{N}\Gamma_k \ln(\mathbf{r} - \mathbf{r}_{ik}) \quad (3)$$