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Pulsating flow of an incompressible micropolar fluid between permeable beds with an inclined uniform magnetic field

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ABSTRACT

In this paper, we investigate the pulsating flow of an incompressible and slightly conducting micropolar fluid between two homogeneous permeable beds in the presence of an inclined uniform magnetic field. The flow between the permeable beds is assumed to be governed by Eringen's micropolar fluid flow equations and that in the permeable beds by Darcy's law with the Beavers–Joseph slip conditions at the fluid–permeable bed interfaces. It is assumed that a uniform magnetic field is applied at an angle θ with the y-axis. The equations are solved analytically and the expressions for velocity and microrotation are obtained. The effects of the magnetic parameter and the other material parameters are studied numerically and the results are presented through graphs.

1. Introduction

Pulsating flow is a periodic flow that oscillates around non-zero mean value. The severity of pulsating flow depends on pulsation amplitudes, frequency and waveform. A complete treatment of the fluid dynamics of steady and pulsatory flow with emphasis on basic mechanics, physics and applications can be seen in [1]. Carpinlioglu and Gundogdu [2] presented a review on the pulsatile pipe flow studies directing towards future research topics. Pulsating flow is generally encountered in natural systems such as human respiratory, circulatory and vascular systems and in engineering systems such as internal combustion engines, thermoacoustic coolers, Stirling engines, bio-reactors and MEMS microfluidic engineering applications.

The laminar flows in channels with permeable walls have gained considerable attention because of their applications in modelling pulsating diaphragms, filtration, and grain regression during solid propellant combustion. Wang [3] studied the pulsatile flow in a porous channel. Vajravelu et al. [4] studied the pulsatile flow between permeable beds. Malathy and Srinivas [5] studied the pulsating flow of a viscous, incompressible, Newtonian fluid between permeable beds under the influence of transverse magnetic field. Ramakrishnan and Shailendhra [6] studied hydromagnetic steady flow through uniform channel bounded by porous media. Avinash et al. [7] studied the pulsatile flow of a viscous stratified fluid of variable viscosity between permeable beds. Iyengar and Punnamchandar [8,9] studied pulsating flows of couple stress fluid and micropolar fluid between two parallel permeable beds in the absence of magnetic effects.

To the extent the present authors have surveyed the pulsatile flow of an incompressible micropolar fluid between two permeable beds with an inclined uniform magnetic field has not been studied so far. The micropolar fluid model introduced by [10] can be used to explain the flow of liquid crystals with rigid molecules, magnetic fluids, biological fluids, lubricants, polymeric additives, geomorphological sediments, colloidal suspensions, haematological suspensions etc. In such fluids the micro-elements possess both translational and rotational motions. The interaction of the velocity field and microrotation field can be described through new material constants in addition to those of a classical Newtonian fluid. Eringen's micropolar fluid model includes the classical Navier–Stokes equations as a special case and can cover both the theory and applications, many more phenomena than the classical model can. Extensive reviews of the theory and applications of micropolar fluids can be found in the books by [11,12].

In this paper, we study the flow of an incompressible micropolar fluid between permeable beds with an inclined uniform

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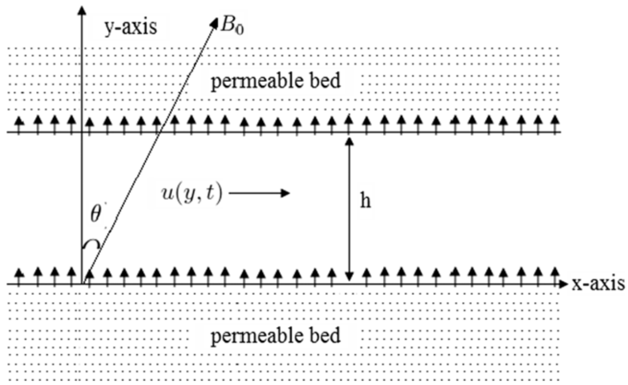

Fig. 1. Flow diagram.

magnetic field. The flow is assumed to be driven by an unsteady pulsating pressure gradient. The flow through the permeable beds is assumed to be governed by Darcy's law and the flow between the permeable beds by Eringen's micropolar fluid flow equations. The Beavers–Joseph (BJ) slip boundary conditions are used at the interfaces of the permeable beds [13]. The equations are solved analytically and the expressions for velocity and microrotation are obtained. The effects of micropolar parameters, Hartmann number, angle of inclination of the uniform magnetic field, porosity parameter, frequency parameter and steady and oscillatory components of the pressure gradient on the velocity and microrotation are studied numerically and the results are presented through graphs.

2. Mathematical formulation

The field equations describing a micropolar fluid flow are [10]

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \bar{q}) = 0 \tag{1}$$

$$\begin{aligned} \rho \frac{d\bar{q}}{dt} &= \rho \bar{f} - \nabla p + \kappa \nabla \times \bar{v} - (\mu + \kappa)(\nabla \times \nabla \times \bar{q}) \\ &\quad + (\lambda + 2\mu + \kappa)\nabla(\nabla.\bar{q}) + \bar{J} \times \bar{B} \end{aligned} \tag{2}$$

$$\begin{aligned} \rho j \frac{d\bar{v}}{dt} &= \rho \bar{l} - 2\kappa \bar{v} + \kappa \nabla \times \bar{q} - \gamma(\nabla \times \nabla \times \bar{v}) \\ &\quad + (\alpha_1 + \beta + \gamma)\nabla(\nabla.\bar{v}) \end{aligned} \tag{3}$$

where $\bar{q}$ and $\bar{v}$ are velocity and microrotation vectors respectively. J is the current density and B is the total magnetic field which is the sum of the applied and induced magnetic fields. $\bar{f}$, $\bar{l}$ are the body force per unit mass and body couple per unit mass respectively and p is the pressure at any point. ρ and j are the density of the fluid and gyration parameter respectively and are assumed to be constant. The material quantities (λ, μ, κ) are viscosity coefficients and $(\alpha_1, \beta, \gamma)$ are gyroviscosity coefficients satisfying the constraints

$$\begin{aligned} &\kappa \geq 0; \quad 2\mu + \kappa \geq 0; \quad 3\lambda + 2\mu + \kappa \geq 0; \\ &\gamma \geq 0; \quad |\beta| \leq \gamma; \quad 3\alpha_1 + \beta + \gamma \geq 0. \end{aligned} \tag{4}$$

We consider the pulsating flow of an incompressible, slightly conducting micropolar fluid between two permeable beds. The permeable beds are rigid and homogeneous. Fig. 1 shows the physical model of the problem under consideration. The Cartesian coordinate system is chosen in such a way that the x-axis is taken along the interface of the lower permeable bed and the y-axis normal to it. Let $y = 0$ and $y = h$ represent the interfaces of the permeable beds. The fluid is injected into the channel from the lower permeable bed with a velocity V and is sucked into the upper permeable bed with the same velocity. The permeabilities of lower and upper beds are k_1 and k_2 respectively. The flow in upper and lower permeable beds is assumed to be governed by Darcy's law. The flow between the permeable beds is assumed to be governed by micropolar fluid flow equations. The thickness of the permeable beds is much larger than the width of the channel so that we can directly use Beavers–Joseph condition at the interfaces of the channel. A uniform magnetic field of strength B_o is applied at an angle θ with the y-axis. The induced magnetic field can be neglected in comparison with the applied magnetic field, as magnetic Reynolds number is much less than unity [14,15]. The fluid is driven by a pulsating pressure gradient given by

$$-\frac{\partial p}{\partial x} = \left(\frac{\partial p}{\partial x}\right)_s + \left(\frac{\partial p}{\partial x}\right)_o e^{i\omega t} \tag{5}$$

where $\left(\frac{\partial p}{\partial x}\right)_s$ and $\left(\frac{\partial p}{\partial x}\right)_o$ are steady and oscillatory components of the pressure gradient respectively and ω is the frequency.

Under the assumptions, we have $\bar{q} = (u(y,t), V, 0)$ and $\bar{v} = (0, 0, c(y,t))$. With this, the governing fluid flow equations of the problem in the absence of body forces and body couples are given by

$$\begin{aligned} \rho\left(\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial y}\right) &= -\frac{\partial p}{\partial x} + \kappa\frac{\partial c}{\partial y} + (\mu+\kappa)\frac{\partial^2 u}{\partial y^2} \\ &\quad - \sigma_e (B_0 \cos\theta)^2 u \end{aligned} \tag{6}$$

$$\rho j\left(\frac{\partial c}{\partial t} + V\frac{\partial c}{\partial y}\right) = -2\kappa c - \kappa\frac{\partial u}{\partial y} + \gamma\frac{\partial^2 c}{\partial y^2}. \tag{7}$$

Herein the velocity component $u(y,t)$ is to satisfy the conditions

$$\left.\begin{aligned} \frac{\partial u}{\partial y} &= \frac{\alpha}{\sqrt{k_1}}(u_{B_1} - Q_1) \quad \text{at } y = 0 \\ \frac{\partial u}{\partial y} &= -\frac{\alpha}{\sqrt{k_2}}(u_{B_2} - Q_2) \quad \text{at } y = h \end{aligned}\right\} \tag{8}$$

and the microrotation component $c(y,t)$ is to satisfy the condition

$$c = 0 \quad \text{at } y = 0 \quad \text{and} \quad y = h \tag{9}$$

where σ_e is the electrical conductivity and B_o is the applied magnetic field. $u_{B_1} = u|_{y=0}$ and $u_{B_2} = u|_{y=h}$ are the slip velocities at the interfaces of the lower and upper permeable beds respectively. α is the slip parameter. $Q_1 = -\frac{k_1}{\mu}\frac{\partial p}{\partial x}$ and $Q_2 = -\frac{k_2}{\mu}\frac{\partial p}{\partial x}$ are Darcy's velocities in the lower and upper permeable beds respectively. Eq. (8) represents the BJ slip conditions. Eq. (9) stipulates that the microrotation vanishes at the interfaces of the permeable beds.

In view of the pulsating pressure gradient, let us assume that the velocity and microrotation are in the form

$$u(y,t) = u_s(y) + u_o(y)e^{i\omega t} \tag{10}$$

$$c(y,t) = c_s(y) + c_o(y)e^{i\omega t} \tag{11}$$

where u_s and c_s represent steady parts and u_o and c_o represent the oscillatory parts of the velocity and microrotation respectively.

The following non-dimensionalization scheme is introduced to make the governing equations and the boundary conditions dimensionless.

$$\begin{aligned} &u^* = \frac{u}{V}, \quad u_s{}^* = \frac{u_s}{V}, \quad u_o{}^* = \frac{u_o}{V}, \\ &c^* = \frac{ch}{V}, \quad c_s{}^* = \frac{c_s h}{V}, \quad c_o{}^* = \frac{c_o h}{V}, \quad u^*_{B_1} = \frac{u_{B_1}}{V}, \\ &u^*_{B_2} = \frac{u_{B_2}}{V}, \quad p^* = \frac{p}{\rho V^2}, \quad \omega^* = \frac{\omega h}{V}, \\ &t^* = \frac{tV}{h}, \quad x^* = \frac{x}{h}, \quad y^* = \frac{y}{h}. \end{aligned} \tag{12}$$