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Mathematical modelling of pulsatile flow of blood through catheterized unsymmetric stenosed artery—Effects of tapering angle and slip velocity

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ABSTRACT

In this paper the dynamic mathematical model of blood flow through a stenosed tapered artery in the presence of a catheter has been studied. Blood is modelled as a homogeneous and incompressible couple stress fluid. The analytical solution for the velocity component (v_z) is obtained in terms of Bessel functions of various kinds. The effects of tapering angle (θ), slip velocity (u), catheter radius (r_c), Reynolds number (Re), pulsatile parameter (σ), maximum height of the stenosis (ϵ) and couple stress fluid parameters (β, ω) on physiological parameters such as Impedance and Wall shear stress (at the maximum height of the stenosis) are discussed. The locations of the maximum height of the stenosis and the annular radius which are dependent on both tapered parameter (ζ) and shape parameter (n) are computed. It is identified that the impedance and wall shear stress are increasing as the tapered parameter (ζ) and the couple stress fluid parameters are decreasing while the behaviour is reverse in the case of the radius of the catheter (r_c) and ϵ. It is also observed that the impedance is more in the case of symmetric stenosis. Further the effect of tapering angle on physiological parameters shows that the impedance and wall shear stress are maximum in the case of converging tapered arteries. To validate the results obtained using the present model, the results are compared with the experimental results of Back (1994).

1. Introduction

The heart and circulatory related problems are the major causes for ill health and death in the present world. According to the World Health Organization (WHO) about 17 million deaths in 2008 were heart related. The root cause for cardiovascular disease which is a vascular dysfunction is a consequence of atherosclerosis, thrombosis or high blood pressure, which then compromises organ function. Most notably, the heart and brain can be affected, as occurs in myocardial infarction and stroke, respectively.

The amount of calcium, fatty components and cholesterol which gets deposited on the inner walls of the artery narrows the lumen of the artery which in turn results in restricted flow of blood to various parts of the body. This condition of the artery is known as Atherosclerosis. The flow characteristics will significantly get affected in the vicinity of the narrow arteries. Since blood plays a crucial role in transporting the Oxygen and many physiological fluids to different parts of the body, like transport of urine from kidney to the bladder through the ureter etc., the disturbance in the flow due to stenosis alters the functioning of human body. For diagnosing and treating the heart and blood vessels condition, procedures involving catheters (a long thin flexible tube) are used. One procedure involving catheters called balloon angioplasty is used to treat atherosclerosis. The intro-mission of a catheter into a stenosed artery will alter the flow, impedance, pressure distribution and shear stress. Hence the study of the catheter effects in physiological artery flows is very important as it is also interdisciplinary.

Many earlier researchers [1–3] studied the blood flow through a stenosed artery by considering blood as a Newtonian fluid. Back and Denton [4] obtained the estimates of the wall shear stress and discussed its clinical importance in coronary angioplasty. Back [5] estimated the resistance to the flow by assuming that the blood has Newtonian behaviour. Based on the results obtained by him it can be concluded that even a very small angioplasty guide-wire leads to sizable increases in flow resistance. But it is well known that blood flow in small arteries at low shear rate behaves as a non-Newtonian fluid. Blood was modelled by a micropolar fluid and the effect of stenosis was discussed by Devanathan [6]. Srinivasacharya

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and Srikanth [7] discussed the steady flow of a micropolar fluid through a stenosed catheterized artery. Although there are several models which describe the non-Newtonian nature of the blood, the couple stress fluid introduced by Stokes [8] has special significance. Since it is a polar fluid it accounts for all features related to it. Further the couple stress fluid theory incorporates all the features and effects of couple stresses and results in equations that are similar to the Navier–Stokes equations thereby facilitating a comparison of the results with that of the classical non-polar fluid theory. The foreground of couple stresses is to introduce a size-dependent effect that is not present in the other classical viscous theories. This model has been widely used [9,10] because of its relative mathematical simplicity compared with other models developed for the couple stress fluid. Srikanth and Kebede [11] analysed the pulsatile nature of blood using micropolar and couple stress fluids in the case of multiple stenosis.

In the aforementioned studies the shape of stenosis was considered to be symmetric about the axis as well as radius. But it is understood that stenosis might be non-symmetric while it is growing. Flow through radially non-symmetric stenosis has been analysed by Srivastava and Devajyoti [12,13]. Pielhop et al. [14] conducted experiments using different mixtures of water and glycerin by Time-resolved particle-image velocimetry (TRPIV) measurements combined with synchronous measurements of the static pressure in a highly elastic, axisymmetric stenosed, transparent vessel model at pulsatile flow. The TRPIV measurements show the stenosis to induce a jet through the throat together with complex flow structures like ring vortices. Diana Massai et al. [15] in their study presented a comprehensive analysis of the local haemodynamics within an image-based model of a 51% stenosed internal carotid artery, focused on the influence of the disturbed flow caused by the stenosis on transport and flow-induced activation of platelets. Their results confirm that the presence of stenosis enhances the risk for flow-induced activation of platelets.

There is no doubt that tapering in arteries is a significant aspect of the mammalian arterial system and the formation of stenosis along the tapered wall may alter the flow situation to a great extent. The flow through an asymmetric stenosed tapered artery was discussed by various authors [16,17]. So, the effects of vessel tapering together with the shape of stenosis on the flow characteristics seem to be equally important and hence deserve special attention.

In most of the aforesaid studies, the conventional no-slip boundary condition has been employed. However, a number of studies of suspensions in general and blood flow in particular, as done theoretically by Vand [18], Brunn [19] and experimentally by Bennett [20] have suggested the likely presence of slip (a velocity discontinuity) at the flow boundaries. As per their studies the apparent (effective) viscosity seems to get lowered, as a result of wall slip. Recently, Misra and Shit [21] have developed mathematical models for blood flow through a stenosed arterial segment, by taking a velocity slip condition at the constricted wall. Thus, it seems that the consideration of a velocity slip at the stenosed vessel wall will be quite rational, in blood flow modelling.

The above literature survey is motivating enough for the authors to emphasize the effects of slip velocity, catheterized tapered artery and asymmetric stenosis on physiological parameters such as impedance and wall shear stress. The analysis is carried out on the pulsatile flow of a couple stress fluid.

2. Formulation of the problem

Consider the flow of an incompressible homogeneous couple stress fluid between two coaxial cylinders represented by the artery and catheter. r_c is the radius of the catheter while r_0 is the annular radius. The flow is assumed to be axisymmetric and it is flowing in an asymmetric stenosed artery in the presence of a catheter as shown in Fig. 1.

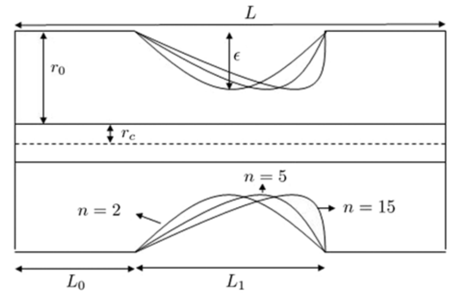

Fig. 1. 2D view of the physical model of the stenosed artery with catheter.

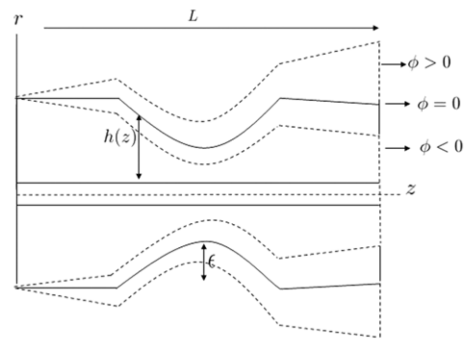

Fig. 2. Tapered stenosed artery with catheter 2D view.

The geometry of the unsymmetric stenosis is expressed mathematically as shown below [16]:

$$h(z) = \begin{cases} (r_0 + \zeta z)\left[1 - \dfrac{\epsilon n^{n/(n-1)}}{(n-1)L_1^n}\left(L_1^{n-1}(z - L_0) - (z - L_0)^n\right)\right], \\ \quad L_0 \le z \le L_0 + L_1 \\ (r_0 + \zeta z) \quad \text{otherwise} \end{cases}$$

where r_0 is the radius of the annular region, L_0 is the length of the artery up to the starting point of the stenosis and L_1 is the length of the stenosis while the total length of the artery segment is L. $n(\ge 2)$ determines the shape of the stenosis (stenosis is symmetric when $n = 2$). ϵ is the maximum height of the stenosis and $\zeta (= \tan\phi)$ is the tapered parameter as shown in Fig. 2. $\phi < 0$, $\phi = 0$ and $\phi > 0$ correspond to converging tapered artery, non-tapered artery and diverging tapered artery respectively.

The location of the maximum height of the stenosis is depending on both n and ζ. The location of the maximum height of the stenosis is obtained from the equation

$$n\,(z - L_0)^{n-1} - L_1^{n-1} + \frac{\zeta}{r_0}\Bigg[(n+1)(z - L_0)^n - 2L_1^{n-1}(z - L_0) + \left(\frac{n-1}{\epsilon\, n^{n/(n-1)}} - \frac{L_0}{L_1}\right)L_1^n + L_0\, n\,(z - L_0)^{n-1}\Bigg] = 0. \quad (1)$$

Further in the case of non-tapered artery i.e., when $\zeta = 0$, the maximum height of the stenosis is found to be located at

$$z = L_0 + \frac{L_1}{n^{1/(n-1)}}. \quad (2)$$

The equations governing the flow of a couple stress fluid are given by

$$\frac{D\rho}{Dt} + \rho(\nabla.\overline{V}) = 0 \quad (3)$$

$$\rho\frac{D\overline{V}}{Dt} = -\nabla P + (\lambda + \mu)\nabla\nabla.\overline{V} + \eta\nabla^2\nabla\nabla.\overline{V} + \mu\nabla^2\overline{V} - \eta\nabla^4\overline{V} + \rho\bar{f} + \frac{1}{2}\nabla \times (\rho\bar{l}) \quad (4)$$