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The turbulent rotating-disk boundary layer

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ABSTRACT

The turbulent boundary layer on a rotating disk is studied with the aim of giving a statistical description of the azimuthal velocity field and to compare it with the streamwise velocity of a turbulent two-dimensional flat-plate boundary layer. Determining the friction velocity accurately is particularly challenging and here this is done through direct measurement of the velocity distribution close to the rotating disk in the very thin viscous sublayer using hot-wire anemometry. Compared with other flow cases, the rotating-disk flow has the advantage that the highest relative velocity with respect to a stationary hot wire is at the wall itself, thereby limiting the effect of heat conduction to the wall from the hot-wire probe. Experimental results of mean, rms, skewness and flatness as well as spectral information are provided. Comparison with the two-dimensional boundary layer shows that turbulence statistics are similar in the inner region, although the rms-level is lower and the maximum spectral content is found at smaller wavelengths for the rotating case. These features both indicate that the outer flow structures are less influential in the inner region for the rotating case.

1. Introduction

A rotating disk draws fluid towards its surface, where a three-dimensional boundary layer forms within which fluid is transported both azimuthally with the disk and outwards in the radial direction. An exact similarity solution for the laminar boundary layer, first reported by von Kármán [1], exists for an infinite disk rotating in an otherwise quiescent fluid, for which the boundary-layer thickness is constant, independent of radius (see Fig. 1 for a definition of the flow field and coordinate system and Appendix A for the corresponding Reynolds-averaged flow equations). The instability of the laminar boundary layer and the laminar–turbulent transition process have been studied extensively experimentally, theoretically and numerically, and some of the most pertinent results are given in Refs. [2–13]. However, only few studies of the turbulent boundary layer on the rotating disk have been reported (for literature prior to 1994 see Littell and Eaton [14] and references therein) despite the fundamental scientific interest in three-dimensional turbulent boundary layers and their technical applications (e.g. rotor-stator systems, see Del Arco et al. [15]). In the following we will denote the turbulent boundary layer on the rotating disk as the von Kármán turbulent boundary layer and abbreviate it as vKTBL. As usual it is a Reynolds number that determines whether the flow is laminar or turbulent and in this case the Reynolds number R is defined as $R = r(\Omega/\nu)^{1/2}$, where r is the radius of the disk at the measurement position, Ω is the rotational speed of the disk, ν is the kinematic viscosity of the fluid. The two-dimensional turbulent boundary layer over a flat plate is abbreviated herein as 2DTBL.

An interesting observation, first pointed out by Lingwood [6], is that the onset of transition for the flow on a smooth disk is reported in the experimental literature to occur within a rather small Reynolds number range. Imayama et al. [11] determined that a fully-developed turbulent boundary layer is established beyond a Reynolds number of approximately 650.

Some early work on the drag exerted by turbulent flow on rotating disks was undertaken by Goldstein [16]. Theodorsen and Regier [17] also performed measurements of drag on “revolving disks” but in addition measured the azimuthal velocity profiles in the laminar, transitional and turbulent regions up to about $R = 2600$. In contrast to the laminar boundary layer the turbulent boundary-layer thickness was shown to increase in the radial direction.

Cham and Head [18] determined the distribution of the radial and azimuthal velocity of the vKTBL with Pitot tube measurements. From their Fig. 15 it is possible to estimate the local flow angle although they did not give it explicitly. In the outer part of the

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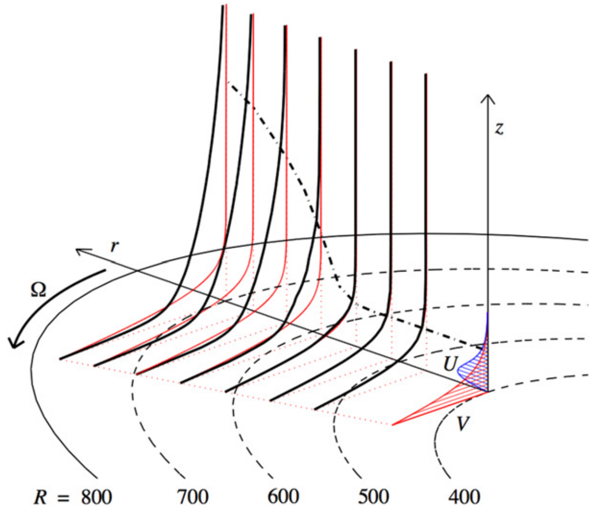

Fig. 1. Definition of the rotating disk geometry and coordinate system. The azimuthal velocity profiles shown in red correspond to the theoretical laminar case, whereas the black are measured profiles. The chain-dotted line indicates the boundary-layer thickness, where the velocity has decreased to 5% of the disk velocity at the same radial position. The deviation from the theoretical laminar distribution can start to be observed above $R = 550$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

boundary layer, i.e. where the azimuthal velocity is less than 20% of the velocity of the disk at the same radius, the flow angle with respect to the azimuthal direction can be estimated to be 20°–30° depending on the Reynolds number, however the angle decreases substantially towards the wall. Streamline visualization behind small roughness elements gave angles of about 11° but according to the authors should be viewed with caution since the roughness elements still protruded through the viscous sublayer. More recent experiments on the vKTBL have been performed at Stanford University [14,19] and in Japan [20] using hot-wire anemometry. Both experiments use rotating disks with a large diameter of 1000 mm, which is advantageous to obtain high Reynolds numbers but still a relatively large viscous length scale $\ell_*[=\nu/v_\tau]$ ($v_\tau = \sqrt{\tau_{w,\theta}/\rho}$ is the friction velocity, and $\tau_{w,\theta}$ the wall shear stress in the azimuthal direction and ρ is the fluid density). This can be understood from the definition of ℓ_*, which can be written as:

$$\ell_* = \frac{\nu}{v_\tau} = \sqrt{\frac{2}{c_f}}\frac{\nu}{\Omega_z r} = \sqrt{\frac{2}{c_f}}Re^{-2}r \qquad (1)$$

where the skin friction coefficient $c_f[=2v_\tau^2/(\Omega_z r)^2]$ is expected to be a weak (decreasing) function of the Reynolds number. Then Eq. (1) shows that for a given Reynolds number the viscous length scale increases linearly with the radius. These groups both published their first results in 1994 but it seems that they were not aware of each other's work since there were no cross references. One result from Itoh and Hasegawa [20] worth mentioning here is their determination of the limiting flow angle at the disk, which was found to be close to, but slightly larger than the angle (11°) obtained by Cham and Head [18].

A large-eddy simulation of the vKTBL was reported by Wu and Squires [21] at $R \approx 800$. They used different turbulence models and in the outer region the results were fairly independent of the model and compared well with the measurements in Ref. [14]. However, in the near-wall region their results on the turbulence statistics were inconclusive since different turbulence models gave different results. Littell and Eaton [14] did not measure close to the wall so in that region comparisons were made with 2DTBL simulations and quite large discrepancies were observed in the near-wall region for the reported data.

An important quantity for turbulent boundary layers is the wall shear stress, or rather the friction velocity, which is the appropriate scale for the mean velocity and Reynolds stresses. Different methods of determining the friction velocity were discussed in Alfredsson et al. [22] and for 2DTBL Nagib et al. [23] suggested that the "oil film interferometry technique is the most reliable method for accurate and direct measurement of mean skin friction (∼1.5%)". However, this technique is not practicable on the rotating disk and instead one has to rely on measurements of the velocity in the viscous sublayer to determine the velocity gradient at the wall. Such velocity measurements were performed by Itoh and Hasegawa [20], both for the azimuthal and the radial directions, whereas in Littell and Eaton [14] the friction velocity (in the azimuthal direction) was based on the Clauser plot technique. However, Nagib and Chauhan [24] have shown that the von Kármán constant κ, which is one of the coefficients for the logarithmic law in the turbulent boundary layer, may take different values depending not only on Reynolds number but also on the flow system (e.g. different values are suggested for boundary layers, pipes and channels) and therefore the Clauser plot technique cannot be used as an independent technique to determine the wall shear stress.

While velocity measurements in the viscous sublayer are, therefore, preferred, hot-wire measurements generally give errors in the near-wall region because of heat transfer from the hot-wire probe to the wall. However, measurements of the azimuthal vKTBL velocity profile give a maximum azimuthal velocity at the wall (assuming that the hot-wire probe is fixed in the laboratory frame of reference). This means that heat conduction from the wall to the probe becomes relatively small compared with heat convection, and direct measurement of the skin-friction velocity using hot-wire anemometry becomes possible as does evaluation of accurate turbulence statistics in the near-wall region. Furthermore, Alfredsson et al. [25] showed that the cumulative distribution function (CDF) of the velocity in the near-wall region shows similarity close to the wall for 2DTBL flows. It is proposed here that such a similarity also holds for the vKTBL (see Appendix B). This proposition, which is tested here, also allows evaluation of heat-transfer effects and how well the wall position has been determined.

The measurements by Itoh and Hasegawa [20] are reported for three Reynolds numbers corresponding to $R = 632$, 775 and 1000. The sensor lengths of the hot-wire in viscous units are 19, 30 and 45, respectively, for the three cases. Turbulence intensity measurements are shown from approximately $z^+ = 5$ to the boundary-layer edge and show the expected near-wall peak around $z^+ = 15$ although its amplitude decreases with increasing R. In contrast, for the 2DTBL the peak in the near-wall turbulence intensity increases with Reynolds number, which suggests that the vKTBL observation is a spatial-averaging effect. In Littell and Eaton [14] no turbulence measurements are presented in the near-wall region; the closest wall position is above approximately $z^+ = 100$. Results of higher moments and spectra were not presented in either of these studies.

The aim of the present study is to evaluate the turbulence statistics of the vKTBL and compare them with the 2DTBL especially in the near-wall region, for which only the experimental results of Itoh and Hasegawa [20] have been reported to date. To do so, direct determination of the skin-friction velocity (using hot-wire anemometry measurements of the velocity distribution in the viscous sublayer) has been one of the primary foci. It will be shown that close to the wall the boundary-layer statistics are comparable with those of 2DTBL whereas the outer region shows distinct differences. Comparisons with the other vKTBL studies are also made, and these are complemented by measurements of higher moments (skewness and flatness factors) as well as spectral maps of the azimuthal velocity.