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Slow motion of spherical droplet in a micropolar fluid flow perpendicular to a planar solid surface

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ABSTRACT

The Stokes axisymmetrical flow problem of a viscous fluid sphere moving perpendicular to an impermeable bounding surface within a micropolar stagnant fluid as well as the related problem of a micropolar fluid sphere moving perpendicular to an impermeable planar surface within a stagnant viscous fluid are considered. The fluids are considered to be incompressible, and the deformation of the fluid particle is neglected. A general solution is constructed from fundamental solutions in both cylindrical and spherical coordinate systems. As boundary conditions, continuity of velocity, continuity of shear stress and the spin–vorticity relation at the droplet surface are applied. Also the no-slip and no-spin boundary conditions are used at the impermeable plane surface. A combined analytical-numerical procedure based on collocation technique is used. The drag acting, in each case, on the fluid particle is evaluated with good convergence. Numerical results for the normalized hydrodynamic drag force versus the relative viscosity, relative separation distance between the particle and wall, micropolarity parameter (a viscosity ratio characterizing micropolar fluids) and spin parameter (a non-dimensional scalar factor relating the microrotation and vorticity at the droplet surface) are presented both in tabular and graphical forms. The results for the drag coefficient are in good agreement with the available solutions in the literature for the limiting cases.

1. Introduction

Several non-Newtonian fluid models have been proposed to describe fluids of microstructures. A physically relevant model that has many applications is the micropolar fluid which was introduced by Eringen in the middle of 1960s [1,2]. Physically, a micropolar fluid is a suspension of rigid, randomly oriented, particles [3]. In micropolar fluids, individual particles can rotate independently from the rotation and movement of the fluid as whole and their deformation is neglected. Therefore, new variables which represent angular velocities of fluid particles and new equations governing this variable should be added to the conventional model.

The micropolar theory can be applied in an increasingly significant number of cases in various scientific fields. Listed among them are the study of lubricating fluids in bearings in lubrication theory [3–5]. A micropolar fluid model also successfully describes granular flow [6–9]. Actually, granular flows are flows which have micro-structure and rotation of particles. Hayakawa [6] has reported that the theoretical calculations of certain boundary value problems are in agreement with relevant experimental results of granular flows. Eringen's micropolar model includes the classical Navier–Stokes equations as a special case, but can cover, both in theory and applications, many more phenomena than the classical model. A comprehensive review of micropolar fluid theory and some of its applications is presented in the textbook by Łukaszewicz [3].

The theoretical study of the movement of fluid droplets in a second immiscible fluid has grown out of the classical study of Hadamard [10] and Rybczynski [11] of the translation of a fluid sphere in an immiscible fluid. This problem is also treated by Happel and Brenner in their book [12]. Hetsroni and Haber [13] used the method of reflection to solve the problem of a single droplet submerged in an unbounded viscous fluid of different viscosity. In practical situations of Stokes flow, particles or droplets are not isolated, and the surrounding fluid is externally bounded by solid or permeable walls. The hydrodynamic interaction between particles or droplets and a wall is of interest for various applications; e.g. sedimentation [14], motion of blood cells in an artery or vein [15,16], and suspensions processing [17]. Therefore, it is required to determine whether the presence of neighboring boundaries considerably affects the movement of a particle or droplet.

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The interactions of a particle or a droplet with walls depend on the particle shape, orientation, and position as well as the geometry of the containing walls. Using spherical bipolar coordinates, Bart [18] examined the motion of a spherical fluid drop settling normal to a plane interface between two immiscible viscous fluids. Wacholder and Weihs [19] also utilized bipolar coordinates to study the motion of a fluid sphere through another fluid normal to a no-slip or free plane surface; their calculations agree with the results obtained by Bart in these limits. The parallel motion of a nearly spherical drop between two channel walls in a quiescent fluid was considered by Shapira and Haber [20] using the method of reflections. Approximate solutions for the hydrodynamic drag force exerted on the droplet were obtained, which are accurate when the drop-to-wall spacing is not small.

The boundary collocation method has been used by many authors to solve flow problems in viscous fluids. Gluckman et al. [21] developed a truncated series boundary collocation method to study the unbounded axisymmetric multispherical Stokes flow. The theoretically-predicted drag results are in good agreement with experimentally measured values. Later, Leichtberg et al. [22] extended the work of Gluckman et al. [21] to bounded flows for co-axial chains of spheres in a tube. Ganatos et al. [23,24] modified the collocation series solution techniques to investigate the Stokes flow of perpendicular and parallel motion of a sphere between two parallel plane boundaries. Boundary-collocation techniques are used to examine the parallel and perpendicular motions of spherical drops moving near one plane wall and between two parallel plates as a function of drop size and viscosity ratio [25,26]. The solutions of their work agree well with a previous study on the motion of rigid spheres [23,24] when the drop-to-medium viscosity ratio tends to infinity.

All results cited above concern viscous fluids. For micropolar fluids, Ramkissoon [27,28] has studied the Stokes flow of a micropolar fluid past a Newtonian viscous fluid sphere and spheroid. The two related problems of the flow of a viscous fluid past a fluid sphere which has a micropolar fluid inside it and the flow of a micropolar fluid past a viscous fluid drop are discussed by Niefer and Kaloni [29] with non-zero spin boundary condition. Various spin boundary conditions have been proposed in the literature [30–33]. The resistance force exerted on a solid sphere moving with constant velocity in a micropolar fluid with a nonhomogeneous boundary condition for the microrotation vector was calculated by Hoffmann et al. [34]. The problem of Stokes axisymmetrical flow of an incompressible micropolar fluid past a liquid droplet-in-cell models has been investigated analytically by [35]. Sherief et al. [36,37] discussed the Stokes axisymmetrical flow caused by a sphere translating in a micropolar fluid perpendicular to a plane wall and between two parallel plane walls at an arbitrary position from them. Although, many authors, as mentioned above, discussed the movement of solid spherical or non-spherical particles or droplets in micropolar fluid flow problems, the interaction problems between particles and walls attracted the attention of low number of authors. This motivated us to consider the present study.

In this paper, a combined analytical-numerical solution to two related problems involving micropolar fluids is presented. One is of a viscous fluid sphere immersed in a micropolar fluid and moving away from an impermeable plane wall in a direction normal to the wall. The second is the reverse context of a micropolar fluid sphere immersed in a viscous fluid, again as the spherical drop moves away from the wall. The underlying assumption is made that surface tension is sufficiently strong to prevent deformation but otherwise no account of surface tension is made. The matching boundary conditions on the fluid sphere are that the velocity is continuous, the shear stress is continuous and that the microrotation proportional with vorticity. Numerical results are obtained by evaluating the solution and applying boundary collocation methods for points on the fluid sphere. The drag force on the translating fluid sphere is evaluated for each case. The effects of the variation of the micropolarity and spin parameters, relative viscosities of the droplet and the ratio of the radius of the droplet to the separation distance (the distance from the center of the fluid to the wall) on the normalized drag force as revealed by numerical studies is shown through figures. For the special case of the classical fluid, our calculations show good agreement with the available solutions in the literature for the corresponding motion of a droplet/particle to a plane wall.

2. Field equations

The equations governing the steady flow of an incompressible micropolar fluid in the absence of body forces and body couples are given by

$$\nabla \cdot \vec{q} = 0, \tag{2.1}$$

$$(\mu + k)\, \nabla^2 \vec{q} + k\, \nabla \wedge \vec{\nu} - \nabla p = \rho\, (\vec{q} \cdot \nabla)\, \vec{q}, \tag{2.2}$$

$$(\alpha + \beta + \gamma)\, \nabla\nabla \cdot \vec{\nu} - \gamma\, \nabla \wedge \nabla \wedge \vec{\nu} + k\, \nabla \wedge \vec{q} - 2k\, \vec{\nu} = \rho\, j\, (\vec{q} \cdot \nabla)\, \vec{\nu}, \tag{2.3}$$

where $\vec{q}$, $\vec{\nu}$ ρ, j and p are the velocity vector, microrotation vector, density, microinertia and the fluid pressure at any point, respectively. μ is the viscosity coefficient of the classical viscous fluid and k is the vortex viscosity coefficient. The remaining constants α, β and γ are gyroviscosity coefficients.

The equations for the stress tensor t_{ij} and the couple stress tensor m_{ij} are defined by the constitutive equations

$$t_{ij} = -p\, \delta_{ij} + \mu\, (q_{i,j} + q_{j,i}) + k\, (q_{j,i} - \epsilon_{ijm}\, \nu_m), \tag{2.4}$$

$$m_{ij} = \alpha\, \nu_{m,m}\, \delta_{ij} + \beta\, \nu_{i,j} + \gamma\, \nu_{j,i}, \tag{2.5}$$

where the comma denotes partial differentiation, δ_{ij} and ϵ_{ijm} are the Kronecker delta and the alternating tensor, respectively.

In the limit where inertial forces are small relative to viscous forces, the two nonlinear terms, namely the convective acceleration, $\rho\, (\vec{q} \cdot \nabla)\, \vec{q}$, in Eq. (2.2) and the corresponding term in Eq. (2.3), $\rho\, j\, (\vec{q} \cdot \nabla)\, \vec{\nu}$ can be neglected. That is, let L be some characteristic length q_0, ν_0 some reference value of $|\vec{q}|$, $|\vec{\nu}|$ respectively, then the smaller the dimensionless parameters

$$N_1 = \frac{\rho\, L\, q_0}{\mu + k}, \qquad N_2 = \frac{\rho\, q_0^2}{k\, \nu_0}, \qquad N_3 = \frac{\rho\, j\, L\, q_0}{\alpha + \beta + \gamma},$$
$$M_1 = \frac{\rho\, j\, L\, q_0}{\gamma}, \qquad M_2 = \frac{\rho\, j\, \nu_0}{k}, \qquad M_3 = \frac{\rho\, j\, q_0}{L\, k}, \tag{2.6}$$

the better will be the approximate solutions of the equations obtained by neglecting the inertia terms. We note that N_1 is the well-known Reynolds number while the other represent the relative importance of rotational viscosities to the inertia terms. Therefore, it is reasonable to accept Eqs. (2.2) and (2.3), after dropping the inertia terms, to be applicable in the case of very slow motion. In the following study, we consider the micropolar field equations (2.2) and (2.3) after neglecting the inertia terms. However, some authors consider the full versions of the micropolar field equations in their analyses.

3. Motion of a viscous fluid sphere in a micropolar fluid normal to a plane wall

In the present mathematical model, we consider the quasisteady axisymmetrical motion of a viscous fluid sphere of radius a and viscosity μ' translating with a constant velocity U_z in a second, immiscible micropolar fluid of viscosities $(\mu, k, \alpha, \beta, \gamma)$ in the