European Journal of Mechanics B/Fluids 43 (2014) 110–119

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Vortex shedding from a two-dimensional cylinder beneath a rigid wall and a free surface according to the discrete vortex method

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A R T I C L E I N F O

Article history:
Received 19 June 2011
Received in revised form
24 June 2013
Accepted 4 August 2013
Available online 19 August 2013

Keywords:
Discrete vortex method
Mesh-free form
Rigid wall
Free surface

A B S T R A C T

We investigated the hydrodynamic characteristics and vorticity fields for flow past a 2D circular cylinder beneath a rigid wall and an air–water free surface. It is of practical importance to generalize the viscous mesh-free DVM to cases of flow past a 2D circular cylinder beneath a rigid wall and a free surface. We adopted the image method, which replaces the rigid wall with image singularities, to study the vortex shedding from a circular cylinder in proximity to a rigid wall. Using the Green function for the unsteady motion of a point vortex under a deformable free surface, we applied a linear free surface effect to investigate the hydrodynamic parameters and vortex shedding generated by a 2D stationary circular cylinder under a free surface.

1. Introduction

Flow past a circular cylinder is perhaps the most popular topic in theoretical, experimental and numerical studies of wake flows [1]. It has long been a subject of interest because of relevance to a large number of practical applications such as pipelines, offshore structures, submarines, and equipment for tidal power generation [2]. Despite its simple geometry, flow past a circular cylinder is associated with rich phenomena and is considered to be a baseline case for many engineering problems.

Several previous studies have focused on flow past a circular cylinder in a free stream. There has been extensive research on vortex shedding from a cylinder in a free stream flow in the past two decades, as reviewed elsewhere [3,4]. Ponta and Aref performed a numerical study on periodic vortex shedding in the wake of a cylinder subjected to forced oscillations at low Reynolds number and found the particular phenomenon of $P+S$ vortex shedding [5]. Young et al. developed a coupled BEM and three-step FEM to simulate laminar vortex shedding in incompressible viscous flows past a circular cylinder using 2D N–S equations [6].

However, the literature on flow around a circular cylinder located in proximity to a rigid wall or a free surface is not as extensive as in the case of an infinite medium. In particular, flow interactions between the body wake and the free surface deformation are closely related to a number of engineering problems, such as hydrofoil operation on a free surface and the flow past ocean platform structures beneath sea level. Calculation becomes complicated owing to deformation of the free surface induced by unsteady vortex elements. Bimbato et al. numerically simulated a 2D viscous incompressible flow around a circular cylinder in the vicinity of solid ground [7]. Prasanth and Mittal investigated the vortex-induced vibration of a pair of equal-sized 2D circular cylinders in tandem and staggered arrangements in a laminar flow regime using FEM [8]. Choi and Lee investigated the ground-effect flow characteristics of an elliptic cylinder near a flat plate experimentally [9]. In recent years, only a few studies have considered interactions between the flow past a cylinder and a deformable free surface. Lee and Daichin studied the interaction between a cylinder located above a free surface and the free surface [10]. The experiment was performed in a small-scale wind/wave tunnel and PIV was used to analyze the flow field. Lin and Huang applied a Lagrangian numerical framework to study 2D free surface flow induced by a submerged moving cylinder [11]. Reichl et al. investigated 2D flow past a cylinder close to a free surface for a Reynolds number of 180 in a numerical approach [2].

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0997-7546/$ – see front matter
http://dx.doi.org/10.1016/j.euromechflu.2013.08.004

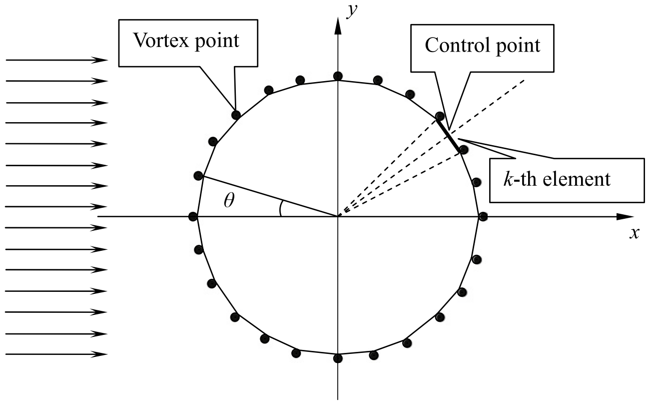

Fig. 1. Schematic of vortex generation and boundary discretization.

Here we investigate the flow field around a 2D circular cylinder located horizontally under a rigid wall and a free surface using the mesh-free discrete vortex method. We also take into account the free surface elevation induced by flow past the cylinder. The hydrodynamic coefficients and vorticity fields are analyzed and compared to a reference case with infinite medium. We focus on three parameters: the Reynolds number $Re = UD/\nu$, where U is the upstream velocity, D is the circular cylinder diameter and ν denotes the kinematic viscosity coefficient; the Froude number $Fr = U/\sqrt{gD}$, where g is acceleration due to gravity; and the gap ratio h/D, where h is the distance between the top of the cylinder and the undisturbed free surface. The main contribution of our study is coupling of known methods in a novel way. Construction of a practical CFD mesh-free tool for investigating hydrodynamic problems is of great interest in engineering.

The remainder of the paper is organized as follows. We provide a brief description of the discrete vortex method and its numerical implementation. Then we study the flow past a 2D circular cylinder. The hydrodynamic parameters and the vortex shedding form are investigated for different gap ratios. Finally, using the Green function for an unsteady point vortex beneath a free surface described by Wehausen and Laitone [12], we analyze the fluid dynamic coefficients, the free surface deformation, and the vorticity field under the influence of a free surface.

2. Fundamentals of the discrete vortex method

We use the discrete vortex method [13,14], which is a Lagrangian numerical scheme for simulating incompressible and viscous fluid flow, to simulate flow past a fixed 2D cylinder. The cylinder is discretized in N elements along the circumference, and N discrete vortices with circulation Γ_i are created at a certain distance from the discretized infinitesimal of the circular cylinder, as shown in Fig. 1. These point vortices are convected and their velocities are assessed as the sum of the free-stream velocity and the induced velocity for the other vortices. On the basis of the boundary condition that the circular cylinder is streamlined, the circulation of each vortex can be calculated.

For 2D viscous and incompressible flow past a circular cylinder at a fixed Reynolds number, the governing equation can be divided into kinematic and dynamic parts. The kinematic part can be expressed as

$$\begin{aligned} \nabla \cdot \mathbf{V} &= 0 \\ \nabla \times \mathbf{V} &= \boldsymbol{\omega} \end{aligned} \tag{1}$$

and the dynamic part is [15]

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = -\mathbf{V} \cdot \nabla \boldsymbol{\omega} + \boldsymbol{\omega} \cdot \nabla \mathbf{V} + \nu \nabla^2 \boldsymbol{\omega}, \tag{2}$$

where t is time and ν denotes the kinematic fluid viscosity. For the 2D problem, Eq. (2) reduces to [16]

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = -\mathbf{V} \cdot \nabla \boldsymbol{\omega} + \nu \nabla^2 \boldsymbol{\omega} = 0. \tag{3}$$

The fluid vorticity ω is expressed as

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \tag{4}$$

where u and v are the velocity components in the x and y directions, respectively. We can introduce the stream function ψ, which satisfies $u = \partial\psi/\partial y$, $v = \partial\psi/\partial x$. Substituting u and v into the vorticity equation, we can obtain the Poisson equation

$$\nabla^2 \psi = -\omega. \tag{5}$$

If the vorticity ω is given, the kinematic equation has the following solution for the boundary value problem:

$$\begin{aligned} \mathbf{V}(\mathbf{r}) = &-\int_R (\boldsymbol{\omega} \times \nabla G)\, \mathrm{d}R \\ &+ \oint_{S_I + S_B} [\nabla G (\mathbf{V}_0 \cdot \mathbf{n}_0) - (\mathbf{V}_0 \times \mathbf{n}_0) \times \nabla G]\, \mathrm{d}S, \end{aligned} \tag{6}$$

where R is the external flow field of the body, and S_I and S_B denote the boundary surface at infinity and the body surface, respectively. $\mathbf{n}_0$ denotes the vector normal to the surface and $\mathbf{V}_0$ is the velocity vector. G is the Green function satisfying the 2D Laplace function

$$G = \frac{1}{2\pi} \log \sqrt{(x - x_0)^2 + (y - y_0)^2}. \tag{7}$$

Suppose that the external boundary surface is circular with an infinite radius. The integral along the external boundary can be degenerated as

$$\begin{aligned} &\oint_{S_I + S_B} [\nabla G (\mathbf{V}_0 \cdot \mathbf{n}_0) - (\mathbf{V}_0 \times \mathbf{n}_0) \times \nabla G]\, \mathrm{d}S \\ &= \oint_{S_I} \frac{1}{2\pi C} [\mathbf{n}_0 (\mathbf{V}_0 \cdot \mathbf{n}_0) + \mathbf{V}_0 (\mathbf{n}_0 \cdot \mathbf{n}_0) - \mathbf{n}_0 (\mathbf{V}_0 \cdot \mathbf{n}_0)]\, \mathrm{d}S \\ &= U. \end{aligned} \tag{8}$$

Thus, Eq. (6) becomes

$$V(r) = -\frac{1}{2\pi} \int_R \frac{\omega_0 \times (r_0 - r)}{|r_0 - r|^2} \mathrm{d}R + U, \tag{9}$$

which is the Biot–Savart law. The crux of the discrete vortex method is numerical simulation of the process of vorticity generation and dispersal using computer-generated pseudo-random numbers [17]. The dynamic vorticity equation (3) can be divided into two parts: the convection current equation

$$\frac{\partial \omega}{\partial t} = -(\mathbf{V} \cdot \nabla)\, \omega \tag{10}$$

and the diffuse equation

$$\frac{\partial \omega}{\partial t} = \nu \nabla^2 \omega. \tag{11}$$

That the fundamental solution to the heat diffusion equation is the Green function:

$$G(x, t) = \frac{1}{\sqrt{4\pi \nu t}} \exp\left(-x^2/4\nu t\right). \tag{12}$$

We find that Eq. (12) is in accordance with a probability density function with mean deviation of zero and standard deviation of σ_x:

$$P(\eta_x, t) = \frac{1}{\sqrt{2\pi \sigma_x^2}} \exp\left(-\eta_x^2/2\sigma_x^2\right), \tag{13}$$