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Asymptotic production behavior in waterflooded oil reservoirs: Decline curves on a simplified model

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ABSTRACT

In this paper we consider a very simplified model of a mature waterflooded oil reservoir and study the asymptotic behavior in time of well's oil production. More precisely, under assumptions on stationary points, we mathematically justify and precise classical decline laws: the oil production rate decreases like $C_1 t^{-\gamma}$ for some $\gamma > 1$ if the nonlinear front velocity vanishes when the oil concentration S is close to vacuum ($\phi(S) = S^\alpha$ with $\alpha > 0$). A more general law is obtained for general vanishing function ϕ at vacuum. It decreases exponentially fast like $C_2 \exp(-t)$ if the nonlinear front velocity does not vanish. Our calculations allow us to express constants C_1, C_2 in terms of physical and geometrical features of the reservoir through PDEs resolution. To the authors' knowledge, this is the first result taking into account space variables. This could be of particular interest for the optimization process of oil production.

1. Introduction

The optimization of oil production for waterflooded mature reservoirs is a crucial industrial problem since field operators try to maximize recovery rates while investment opportunities are often limited. What should be water injectors' and oil producers' location and associated injection and production rates in order to maximize oil production?

A first step in the study of this question is to investigate the asymptotic behavior of a mature oil reservoir, and first to prove that the oil production rate evolves as $t^{-\gamma}$ for some $\gamma > 1$, a classical law. Complete models of oil reservoirs are too complex to be handled so that a first step is to study completely a very simplified model. More complex models will be tackled in forthcoming papers. Note that decline curve analysis has been developed for a long time from a formal point of view for instance in [1–5]. More precisely the empirical Arps decline equation represents the relationship between the oil production rate and time for oil wells during the pseudo steady-state period where S is the oil saturation in the reservoir at time t and S_0 is the initial oil saturation and a, b are two constants. The main hypothesis, made in the previous cited paper, is to assume that the oil rate versus time curve has almost constant loss ratio (see [1, p. 233]) namely

$$\frac{d}{dt}\left(\frac{S}{dS/dt}\right) = -b.$$

Integrating twice, this gives

$$S = S_0\,(a + bt)^{-1/b},$$

where a denotes the loss ratio at time $t = 0$ and $0 \le b \le 1$. If $b = 0$, we get an exponential decreasing decline curve namely

$$S = S_0 \exp(-at).$$

The case $b = 1$ is called a harmonic decline curve. Many published papers have tried to interpret the Arps decline equation looking at the expression of a and b in terms of the available physical parameters. To the authors' knowledge, investigations in physical papers such as [1–5] do not take into account space variables. This is the main objective of our paper on a simple model in a two-dimension space linked to some kind of linearization of well-known porous media equations.

We assume that the pressure satisfies a simple harmonic equation with the Dirac masses distribution in a two-dimensional bounded domain

$$-\operatorname{div}(k(x)\nabla P) = \sum_i \alpha_i \delta_{P_i} \qquad (1)$$

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where $\alpha_i > 0$ for sources and $\alpha_i < 0$ for sinks, and where $k(x) > 0$ is a given smooth function. Let $\mathcal{S}_o$ be the set of sources and $\mathcal{S}_i$ be the set of sinks. The velocity is given by the following Darcy law:

$$V(x) = -k(x)\nabla P(x). \tag{2}$$

The Buckley–Leverett equation is a transport equation used to model two-phase flow in porous media; see for instance [1]. The Buckley–Leverett equation or the Buckley–Leverett displacement can be interpreted as a way of incorporating microscopic effects due to capillary pressure in two-phase flow into Darcy's law. We also assume that the oil concentration follows a nonlinear Buckley–Leverett type that means a transport equation used to model two-phase flow in porous media. It has the form

$$\partial_t S + \phi(S)V \cdot \nabla S = 0 \tag{3}$$

where $\phi(S)$ is a nonlinear function (see for instance [6]). For example, the nonlinear function has the form

$$\phi(S) = S_+^\alpha \tag{4}$$

for some $\alpha > 0$. We must add boundary conditions on P and S, for instance

$$P = 0, \qquad S = 0 \quad \text{on } \partial\Omega \tag{5}$$

and initial condition on S namely

$$S|_{t=0} = S_0. \tag{6}$$

Let $\sigma > 0$ be small enough. Assuming that oil density is equal to one and S denotes the reduced oil saturation, we introduce the oil production rate of the well P_i by

$$E_i(t) = -\int_{C(P_i,\sigma)} \psi(S)V \cdot n, \tag{7}$$

integral over the circle of center P_i and radius σ, where $\psi' = \phi$.

Note that (1)–(4) is a severe simplification. More realistic equations are nonlinear and the coefficient k depends on the solution S itself. However (2) can be seen as a "limit equation" as S goes to 0. By this way, Eqs. (1)–(2) give the velocity V and the saturation S may be computed through the transport equation (3).

Stationary points associated with V, namely points Q where $V(Q) = 0$, play a crucial role in the analysis. We will say that a stationary point is non-degenerate if $dV(Q) \neq 0$.

Readers interested in mathematical studies on more complex porous media systems with Dirac masses are referred for instance to [7]. Using the fact that the system may be seen as a decoupled system, we will prove the following result.

Theorem 1.1. *Let Ω, k, α_j and P_j ($j = 1, \dots, n$ with sources and sinks) be such that there exists a unique solution P of (1) satisfying the boundary conditions (5) $_1$ with a finite number of stationary points in $\overline{\Omega}$, namely points Q such that $V(Q) = 0$. We consider physical situations where all the stationary points are non-degenerate namely satisfy $dV(Q) \neq 0$.*

Assume S_0, ϕ be such that there exists S a solution of (3)–(4) on $(0, +\infty)$ satisfying the boundary conditions (5) $_2$ and the initial condition (6) with V given before through (2).

- *If $\phi(0) = 0$ and if ϕ is a strictly increasing function closed to 0, then the oil production rate (7) of well P_i, for a small enough fixed σ, behaves, for a time t large enough, as*

  $$E_i(t) \sim C_1 t^{-1}\phi^{-1}(C_2 t^{-1}).$$

  In particular the oil production rate behaves like

  $$E_i(t) \sim C_3 t^{-1-1/\alpha} \quad \text{if } \phi(S) = S^\alpha \text{ with } \alpha > 0.$$

- *If $\phi(0) \neq 0$, the oil production rate decreases exponentially fast namely*

  $$E_i(t) \sim C_1 \exp(-C_2 t).$$

Remark. In the next section we will prove this theorem in the case $\phi(0) = 0$, with the case $\phi(0) \neq 0$ being more simple since in this case, we do not have to take care of the singular points. Note that for $\phi(S) = S^\alpha$ with $\alpha > 0$, the oil production rate mentioned in the abstract corresponds to $\gamma = 1 + \alpha$ and C_1 given in (11) calculated through PDEs resolution.

Remark. Remark that closed circulation areas would not play any role since they do not evolve in time and the velocity is regular, divergence free and derived from a gradient far from sources and sinks.

Remark. Note that Eqs. (1) and (3) are decoupled. More precisely existence results linked to (1)–(2) and the appropriate boundary condition (5) related to P may be found in [8] or in [7]. We then have to solve (3) using the transport velocity $V = -k\nabla P$. Note that V is regular enough far from all P_i.

Remark. Note that we have considerably simplified the system assuming that the permeability coefficient k does not depend on the saturation. The equation for the pressure is therefore more simple and the flux associated with the saturation is therefore convex. Usually the flux for the Buckley–Leverett model is a non-convex function because of the dependency of k with respect to the saturation.

Well posedness of the system. Assuming the conductivity regular near wells and sinks, the solution P may be split as $P = P_{\text{singular}} + P_{\text{regular}}$ where the singular part is given by the fundamental solution for a homogeneous porous medium. More precisely

$$P_{\text{singular}} = \sum_{i=1}^{n} P_{i,\text{singular}},$$

where $P_{i,\text{singular}}$ is given by

$$P_{i,\text{singular}}(x) = -\frac{\alpha_i}{2\pi k(x_i)} \ln|x - x_i|$$

the fundamental solution, in two-dimensional whole space, of

$$-\text{div}(k(x_i)\nabla q_i) = \alpha_i\delta_i; \quad i = 1, \dots, n$$

where x_i is the location of the point sources or sinks. The other quantity P_{regular} is given by

$$P_{\text{regular}} = \sum_{i=1}^{n} P_{i,\text{regular}}$$

through the following PDEs in Ω:

$$-\text{div}(k(x)\nabla P_{i,\text{regular}}) = \text{div}\left((k(x) - k(x_i))\nabla P_{i,\text{singular}}\right),$$

with the boundary condition

$$P_{i,\text{regular}} = -P_{i,\text{singular}} \quad \text{on } \partial\Omega.$$

This decomposition has been introduced in [9] and used in [8] to show the well-posedness of such system with P_{regular} in $H^1(\Omega)$ assuming some Hölder continuity of permeability at points x_i. Note that if k is smooth enough, the velocity $V = -k\nabla P$ is smooth outside the Dirac masses distribution.

We then have to solve (3) using the transport velocity $V = -k\nabla P$. Along the characteristics associated with the velocity field V, S satisfies

$$\partial_t S + \phi(S)\partial_s S = 0.$$

Classical regularization approximation (vanishing viscosity method), see [7,6], ensures the solvability for S. With the solution S being a sequence of shocks and smooth parts ending with a relaxation wave. The method of characteristics will be used after a long-time to be prescribed to characterize the large time behavior of S at a point x belonging to $C(P_i, \sigma)$ and therefore prove the theorem.