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Saturation front evolution for liquid infiltration into a gas filled porous medium with counter-current flow

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ABSTRACT

The infiltration of liquid into a gas saturated porous network is investigated. Particular attention is paid to the situation in which a pressure gradient in the porous medium drives a gas flow upwards, while a more dense liquid infiltrates down into the reservoir due to gravity. There are two flows in opposite directions. A model is proposed, based upon a compressible gas phase and an incompressible liquid phase. The volume fluxes in each phase are assumed to be governed by Darcy type flow laws, modified to include the permeability caused by both the solid matrix and the impeding of the gas flow by the liquid phase. Isothermal flows are examined in the absence of phase changes. The proposed model is an extension of the traditional Buckley–Leverett model and is used to consider a variety of flows, including carbon sequestration in a porous medium below the seabed and rainfall infiltration into a lava dome.

1. Introduction

Liquid infiltration into a gas filled porous medium will be investigated. A background pressure gradient will be applied across the porous medium, which drives the gas upwards. The imposed pressure gradient will be insufficient to overcome the effect of gravity on the heavier liquid, setting up counter-current fluid flows as the liquid descends and the gas rises. The effect of the pressure gradient on the gas will be investigated (see Fig. 1).

The infiltration of a fluid into a porous medium is an important process which occurs in many geophysical and industrial situations, including liquid infiltration into soils [1,2], heat exchangers [3,4] and filtration processes [5] as well as biological processes such as fluid flow in the lungs [6,7]. Generally, when such a fluid infiltrates the porous media, it displaces a second fluid that was occupying the void spaces. This displacement can take place by one of two distinct mechanisms. One mechanism is when both fluids completely saturate distinct adjacent regions (on the macroscale) with a mobile interface between the two regions. The second mechanism is that one fluid can displace only a proportion of the second fluid at the macroscale. In this case the fluids co-exist in a state of partial saturation. Of the latter type, Buckley and Leverett [8] began the study of partial saturation of a porous medium and employed the additional assumptions that the process is isothermal and that both fluids are incompressible. The assumption of incompressible fluids is best suited to the case where one liquid is infiltrating a reservoir initially saturated with a second liquid. An example of this process is sea water seeping into an oil reservoir. With these assumptions a single partial differential equation, now called the *Buckley–Leverett equation*, governs the flow of both fluids. This is a hyperbolic partial differential equation for the saturation of the void-spaces by one of the fluids. Under these theoretical simplifications this equation possesses solutions corresponding to a sharp saturation front which propagates into the porous medium. This analysis has been subsequently reviewed by many authors including Bear and Bachmat [9] and Kaviany [10].

In addition to gravity, capillary suction can have a significant affect on liquid infiltration. Richards [11] investigated the influence capillarity has on liquid infiltration in soils, and the equation derived is now known as *Richards' equation*. In the derivation of Richards' equation it is assumed that the liquid infiltration is driven by capillarity and gravity, while the second fluid phase occupying the unsaturated void-spaces is largely inert and does not affect the evolution. Richards' equation has been widely studied for its practical applications and also as an example of degenerate parabolic partial differential equations [12,13].

If the problem of liquid infiltrating a pressurized gas-saturated porous medium is investigated, then the assumption of incompressibility may be justified in the liquid. However, due to the pressurization of the reservoir, the gas should be modelled as

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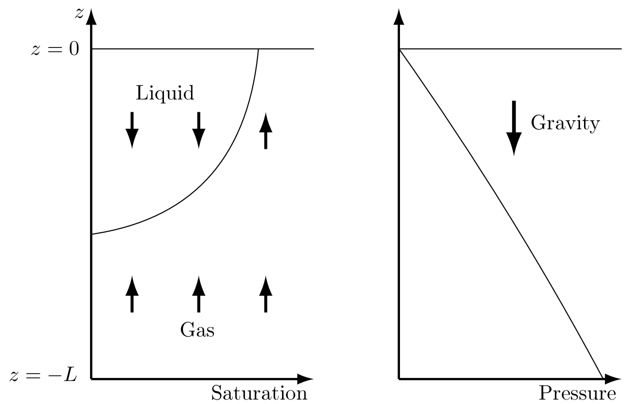

Fig. 1. A schematic diagram of the problem of interest showing liquid infiltrating into a gas filled porous medium. A counter-current flow is established due to phase density differences, gravity and an existing background pressure gradient.

compressible. Therefore generalizations of the models of Buckley and Leverett [8] and Richards [11] are sought, in which one of the fluids present is compressible. Interactions between the different fluid phases occur due to the pressure gradients presented. Our work is motivated by situations in which a liquid is infiltrating a gas-filled porous medium. This situation could arise as the result of carbon sequestration (as part of carbon capture and storage schemes), where pressurized carbon dioxide is stored in an exhausted natural gas field [14,15]. Several exhausted gas fields are found under bodies of water (such as the North Sea); it is possible that if the integrity of the reservoir becomes compromised, then seawater may infiltrate the reservoir, releasing carbon dioxide [16,17]. Models of liquid infiltration into porous media are also of relevance to understanding the processes associated with hydraulic fracturing, which attempts to release hydro-carbon based gases trapped within a porous medium by creating new pathways through which the gas can escape by forcing a liquid into the existing pore spaces at very high pressures [18,19].

There has also been recent interest in the interactions of rainfall and volcanic lava domes as the result of a series of dome collapses after periods of intense rainfall at the Soufrière Hills Volcano, Montserrat [20–22]. A high pressure build-up below the surface of the dome, caused by rainwater infiltrating the void spaces of the dome and interacting with the escaping magmatic gases, may contribute to dome collapse [23,24]. In the case of liquid infiltration into both a pressurized porous medium sequestered with carbon dioxide and a lava dome, the pressure gradient across the saturation interface, gravity and the difference in densities between liquid and gas phases may induce counter-current flows, in which liquid falls predominantly due to its weight, whereas the gas is driven upwards by the dominant influence of the pressure gradient. Across many areas of the surface of a lava dome the measured temperatures are high enough to rapidly boil rainwater landing upon it [25]. This continues until sufficient energy has been expended in boiling water to quench the surface temperature to below boiling point [26,23]. For temperatures below the boiling point of water there will be a descending saturation front at which the temperature equals the boiling point, and above which the temperature is less than or equal to the boiling point. In this situation boiling will reduce the volume of liquid in the porous medium, slowing the front advancement.

Thermal effects are also prevalent in carbon sequestration, where the Joule–Thomson effect acts to cool the gas as it is forced through a porous medium [27,28]. In this case the carbon dioxide may be cooled to the extent that the liquid freezes, which would significantly alter the progress of the front. Carbon dioxide dissolution will occur through interactions between liquid water and gaseous carbon dioxide. The resulting aqueous carbon dioxide solution will increase the liquid acidity and (depending on the rock type) produce porosity and permeability reducing mineral precipitates [29,30]. Additionally, depending on the carbon dioxide concentration and the ambient conditions, the viscosity of an aqueous carbon dioxide solution can differ from that of pure water by up to 38% [31]. As a preliminary investigation into counter-current flows driven by a combination of phase density differences, gravity and a background pressure gradient, an idealized problem is considered in a uniformly porous and permeable material, in which all the processes are isothermal and there are no phase changes. However, once our initial model of counter-current flows has been developed, then it can be extended to include these additional effects and be specifically tailored to model either carbon sequestration or rainwater infiltration.

To further simplify the problem, fluid flows will be considered only in one spatial dimension, which will be aligned parallel to gravity. A schematic diagram of the situation of interest is shown in Fig. 1. However, if additional spatial dimensions aligned parallel to the infiltration front are considered, then flows of this type commonly exhibit viscous fingering as an initially planar infiltration front descends and becomes unstable. In the context of rainwater infiltration into a lava dome, the front is unlikely to propagate too far below the surface. Here a one dimensional model is likely to be appropriate as the initially planar front has not had sufficient opportunity to become unstable. In the context of the Buckley–Leverett problem the stability of a planar saturation front has previously been studied by many authors including Tan and Homsy [32], Chikhliwala et al. [33] and Riaz and Tchelepi [34]. However, the stability analysis of the current situation remains an open problem and is worthy of further study.

In Section 2, a system of equations governing the conservation of mass and momentum of a liquid and gas in a partially saturated region is described. Boundary conditions are considered, which naturally give rise to a pressurized upwards gas flow and a counter-current liquid flow. Steady-state solutions and the initial configuration for the gas profile are considered in Section 3, before the resulting system is investigated with and without capillarity in Sections 4 and 5, respectively. Changes to the degree of saturation at the surface and the underlying gas pressure gradient are considered. In the absence of capillarity the regime of small gas pressure gradient is investigated in Section 4.2, and the behaviour simplifies to that which has been previously reported. Finally, Section 6 contains conclusions and analysis resulting from the modelling.

2. Model development

We consider liquid infiltration and descent into the void spaces of an initially completely unsaturated gas-filled porous medium. Across the porous medium there is a vertical pressure gradient, which drives an upward gas flow. A system of coordinate axes is chosen, in which the z-axis is parallel to the pressure gradient and positive in the upwards direction. The porous medium lies between surfaces at $z = 0$ and $z = -H$. Between $z = 0$ and $z = -L$ a pressure difference exists and in an initial dry configuration this is denoted by $\left[P_g\right]$, with a constant pressure $P_{g,0}$ at $z = 0$ and $L < H$. Attention is restricted to the case in which the two fluid phases are immiscible and phase change between the fluids does not occur.

2.1. Field equations

The mass flux of gas per unit cross sectional area of the porous medium $\tilde{\boldsymbol{m}}_g$ is then related to the gas volume flux per cross sectional area of void-space $\tilde{\boldsymbol{v}}_g$, through the relationship

$$\tilde{\boldsymbol{m}}_g = \phi \tilde{\rho}_g \left(1 - s\right) \tilde{\boldsymbol{v}}_g, \quad (1)$$