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Mesh quality assessment based on aerodynamic functional output total derivatives

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ABSTRACT

Our purpose is to develop a new goal oriented method based on the total derivative of the goal with respect to (w.r.t.) volume mesh nodes. The asymptotic behavior of this derivative as the characteristic cell size tends to zero is first studied. This behavior is assessed using numerical simulations on a hierarchy of meshes. Goal oriented criteria of mesh quality are then proposed based on the same derivative and the local characteristic cell length. Their relevance is assessed using several families of parametrized meshes. The criterion succeeds in sorting the better meshes for goal evaluation from the worse. Finally a local mesh adaptation strategy is proposed and validated. All demonstrations are done for 2D structured meshes with finite-volume schemes and cell-centered approach in the case of Eulerian flow computations.

1. Introduction

In aeronautical CFD, engineers require accurate predictions of the forces and moments but they are less concerned with flow-field accuracy. Hence, the so-called "goal oriented" mesh adaptation strategies have been introduced to get satisfactory values of functional outputs at an acceptable cost, using local node displacement and insertion of new points rather than mesh refinement guided by uniform accuracy. Most often, such methods involve the adjoint vector of the function of interest.

The objective of this study is three-fold: we first study the asymptotic behavior of the total derivative of the goal w.r.t. volume mesh coordinates as characteristic cell size tends to zero (Section 2). This asymptotic behavior is verified on a hierarchy of meshes (Section 5). We then try to qualify the meshes that are well suited for the computation of J (the output of interest) based on one scalar indicator and to derive a corresponding local mesh refinement indicator, both global and local criteria being based on the previously mentioned total derivative of the goal (denoted J) w.r.t. the volume mesh coordinates (denoted X). Until now the Venditti and Darmofal method is the major reference on the last subject for finite-volume methods [1–3]; it has been applied by many authors but has the drawback to require two levels of meshes. For finite element methods, many goal oriented mesh adaptation methods have been developed since the 1990s. Important contributions include the articles of Johnson and co-workers [4–6], Giles and co-workers [7], Prudhomme and Oden [8], Larson and Barth [9], Machiels et al. [10], Hartmann and co-workers [11–13] and Alauzet, Dervieux and co-workers [14]. The search for a criterion using the adjoint vector on a unique level of mesh was rarely considered in the literature. However we can notice the contribution of Dwight [15,16] in which only one level of mesh is necessary but is limited to the classical Jameson et al. numerical scheme [17].

1.1. State of the art on goal oriented mesh adaptation for finite volume schemes

A recent detailed state of the art about output-based error estimation and mesh adaptation can be found in the review by Fidkowski and Darmofal [18]. This article covers both finite-element and finite volume methods. Here, a short presentation of classical adaptation methods for finite-volume schemes is made.

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Nomenclature

AoA	Angle of attack
$\mathcal{B}$	Linear interpolation operator in the reference fine mesh
c	Chord of airfoil
C	Computational space $C = [0, 1]^2$ (resp. $C = [0, 1]^3$) in 2D (resp. 3D)
CD_p, CD_w, CD_{sp}	Pressure drag coefficient, wave drag coefficient and spurious drag
ds_{ij}	Surface element attachable to the point X_{ij}
dX, dXC	Admissible mesh variations and regular function such that $dX_{ij} = dXC(X_{ij})\ \forall i, j \in \{1, N_i\}\{1, N_j\}$
D	Physical space $D \subset \mathbb{R}^2$ (resp. $D \subset \mathbb{R}^3$) in 2D (resp. 3D)
$\mathcal{D}_{(X_{ij},L)}$	Disk of radius L centered in X_{ij}
$\vec{e}_\infty$	Unit vector tangential to the upstream velocity
$F^{(2)}$	Two-point Euler inviscid flux formula
$F^{(4)}$	Four-point Euler inviscid flux formula
F^J	Jameson flux formula
$\mathcal{F}$	Euler inviscid flux density
g_i, g^i	Covariant and contravariant base vectors
g_{ij}, g^{ij}	Covariant and contravariant metric tensors
H, h	Characteristic mesh size of coarse (H) and fine (h) grid
$i, j(, k)$	Mesh indices of a 2D (resp. 3D) mesh
$\bar{i}, \bar{j}(, \bar{k})$	Reduced mesh indices in $[0, 1]^2$ (resp. $[0, 1]^3$)
J	Aerodynamic objective function as a function of volume mesh
J	Aerodynamic function as a function of flow field and volume mesh
$\mathcal{J}$	Aerodynamic function as a function of a vector of design parameters
$k^{(2)}, k^{(4)}$	Artificial dissipation coefficients of Jameson et al. scheme
L	Characteristic size of a mesh deformation
M_∞	Mach number of far-field flow
n_μ	Number of design parameters
$\vec{n}$	Normal vector to solid wall, support of J or outer boundary
N_i, N_j	Number of mesh lines of the structured mesh in each direction
N_W	Size of vectors W and R
p, p_∞	Static pressure and static pressure of far-field flow
$p_a, p_{a\infty}$	Stagnation pressure and stagnation pressure of far-field flow
P	Parametric space $P = [0, 1]^2$ (resp. $P = [0, 1]^3$) in 2D (resp. 3D)
P_a	Mean stagnation pressure over airfoil contour
$P_k, \widetilde{P}^k_{ij}$	Control functions associated to the kth topological direction only and to the kth topological direction and the node (i, j)
$\mathcal{P}(dJ/dX)$	Projection of dJ/dX canceling components orthogonal to function support and solid walls
$\overline{\mathcal{P}(dJ/dX)}$	Spatial mean of $\mathcal{P}(dJ/dX)$
r	Reference variable of the Taylor expansion
R	Finite-volume flux balance
s	Sensor scalar field
$s^{(1)}, s^{(2)}, s^{(3)}$	Sensor fields connected to specific geometrical directions
S	Solid body surface mesh
$S = (S^X, S^Z)$	Interfaces surface vectors
W	Conservative variables (discrete)
w	Continuous flow-field
X	Volume mesh
$\alpha, \beta, \delta, \phi$	Parameters of the mesh families
γ	Specific heat ratio
γ_{ijL}	Discrete estimation of the part of disk centered in the node $X_{i,j}$ that is included in the fluid domain
Γ	Airfoil contour (length $L(\Gamma)$)
θ	Criterion based on $\mathcal{P}(dJ/dX)$
Λ	Adjoint vector of J (J_k) for scheme R
λ	Continuous limit of Λ as the mesh size increases
μ	Vector of design parameter
Φ	Mapping function from $[0, 1]^2$ to $[0, 1]^2$
$\overline{\theta}$	Criterion based on a spatial mean of $\mathcal{P}(dJ/dX)$
$\chi_{N_i,N_j(,N_k)}$	Linear function mapping $[0, 1]^2$ (resp. $[0, 1]^3$) in $[1, N_i] \times [1, N_j]$ (resp. $[1, N_i] \times [1, N_j] \times [1, N_k]$)
Ψ_L	Radial function of support $\mathcal{D}_{(0,L)}$

In a series of three articles [1–3], Venditti and Darmofal have proposed similar formulas for the specific case of finite differences/finite-volume and discrete adjoint, and presented applications to compressible flow computations. Let us define the basic notations employed here for finite-volume CFD computations: W is the flow field (size N_W), X is the volume mesh and R is the residual of the scheme. At steady state, these variables satisfy $R(W, X) = 0$ (set of N_W nonlinear equations to be solved for W). R is supposed to have C^1 regularity w.r.t. its two vector arguments. The method involves two grids: a coarse one of characteristic mesh size H, and a fine one of characteristic mesh size h. The full computation of the flow field and the output of interest on level H is supposed to be affordable, whereas it would be prohibitively expensive on level h. The subscripts h and H will be attached to R, X and W. Finally, W_h^H and λ_h^H represent the coarse-grid flow-field and adjoint vector reconstructed on the fine grid *via* some consistent projection operator. Taylor's expansion of the functional output of interest J_h about the interpolated coarse-grid solution writes:

$$\mathrm{J}_h(W_h, X_h) = \mathrm{J}_h(W_h^H, X_h) + \left(\frac{\partial \mathrm{J}}{\partial W}\bigg|_{W_h^H}\right)(W_h - W_h^H) + \mathcal{O}(\|W_h - W_h^H\|^2).$$

After solving an adjoint-like equation on the fine grid (1), Taylor's expansion of R about W_h^H writes:

$$(\Lambda_h|_{W_h^H})^T \left(\frac{\partial R_h}{\partial W_h}\bigg|_{W_h^H}\right) = -\frac{\partial \mathrm{J}_h}{\partial W_h}\bigg|_{W_h^H} \tag{1}$$

$$\begin{aligned}\mathrm{J}_h(W_h, X_h) &= \mathrm{J}_h(W_h^H, X_h) - (\Lambda_h|_{W_h^H})^T \left(\frac{\partial R_h}{\partial W_h}\bigg|_{W_h^H}\right)(W_h - W_h^H) \\ &\quad + \mathcal{O}(\|W_h - W_h^H\|^2) \\ &= \mathrm{J}_h(W_h^H, X_h) + (\Lambda_h|_{W_h^H})^T R_h(W_h^H) \\ &\quad + \mathcal{O}(\|W_h - W_h^H\|^2). \end{aligned} \tag{2}$$

If the flow computation is not affordable on the fine grid, neither is the solution of Eq. (1) for $(\Lambda_h|_{W_h^H})$. An alternative is to replace this adjoint field by the interpolated coarse-grid adjoint,

$$\mathrm{J}_h(W_h, X_h) \simeq \mathrm{J}_h(W_h^H, X_h) + \underbrace{(\Lambda_h^H)^T R_h(W_h^H)}_{\text{computable correction}} + \underbrace{((\Lambda_h|_{W_h^H})^T - (\Lambda_h^H)^T) R_h(W_h^H)}_{\text{error in computable correction}}.$$

The authors recommend to take $\mathrm{J}_h(W_h^H, X_h) + \Lambda_h^H R_h(W_h^H)$ as the function estimate and adapt the mesh by reducing uniformly the