



## Phase separation in lava flow

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### ABSTRACT

In this paper we propose a model to study the phenomenon of phase separation during lava flow. Lava is considered as a mixture of two incompressible fluids with different density, in that the mass density of the mixture is determined by the concentrations of the two constituents.

We consider as state variables the order parameter, describing the difference in concentration of the fluids, the velocity of the mixture and the absolute temperature. We assume that the order parameter satisfies a Cahn–Hilliard equation, where the chemical potential depends on the velocity and we model lava as a Bingham fluid whose apparent viscosity and yield stress increase exponentially as temperature decreases, according to experimental data. The heat equation provides the evolution equation for temperature. We prove that this model is consistent with the principles of thermodynamics.

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### 1. Introduction

Lava flows are currents of molten or partially molten rocks that run down the slope of a volcano driven by gravity and cool as they flow gradually solidifying until they come to rest. The rheological properties of lavas are of great importance in controlling the dynamics of lava flows. They are strongly temperature dependent and therefore change severely with the cooling of lava after the effusion.

During the early stages, lava is considered as a viscous fluid and its flow is commonly modeled as Newtonian and laminar [1]. But, far from the vent, as a consequence of its cooling, lava behaves as a Bingham fluid, which is characterized by two parameters: the apparent viscosity  $\nu$  and the yield stress  $s$  [2]. The existence of an yield stress can be ascribed to an internal structure which is capable of preventing movement for values of shear stress less than the yield value. Above this value, the internal structure collapses and shearing movement is allowed to occur. Experimental data and theoretical considerations show that temperature has a strong influence on viscosity and yield stress [3,4]. Indeed, as lava flows away from its vent, it loses heat by radiation into the atmosphere and conduction toward the ground and atmosphere, and  $\nu$ ,  $s$  steadily increase.

Lava is a multiphase system which during the cooling process, changes its physical properties. In this paper we focus our attention on the phenomenon of phase separation during lava flow for temperatures above the solidus temperature, that is, the temperature at which lava begins to solidify. After the effusion, as a

consequence of high temperatures, the species composing lava are in a mixed state, but when the temperature goes down a critical value  $\theta_0$ , they may separate into distinct regions. This mechanism, called spinodal decomposition or phase separation, has been developed primarily by Cahn and Hilliard [5] in a metallurgical framework. Later, in recent years, it has been extended to other contexts. In particular, many authors have investigated the spinodal decomposition induced by variations of the temperature or the velocity (see [6–10] and references therein).

For sake of simplicity, we suppose that lava is a mixture of two incompressible fluids with the different density and viscosity.

The phase separation is often described in the framework of phase-field modeling, in that the interface between the two pure phases is not sharp but is regarded as a region of finite width having a gradual variation of different physical quantities. Such a thin layer is called diffuse interface. Following the phase field approach [11], we assume a partial mixing between the two immiscible fluids in the diffuse interface and we introduce a scalar function  $\varphi$ , called order parameter, which allows us to distinguish one phase from the other and which varies smoothly within the diffuse interface.

The paper is organized as follows. In Section 2 we introduce the order parameter  $\varphi$  and we postulate that  $\varphi$  obeys a balance equation of Cahn–Hilliard type, where the chemical potential depends on the velocity in such a way that an increase in the velocity improves the miscibility of the mixture. In Section 3 we introduce the evolution equation for the velocity, modeling lava as a Bingham mixture of two incompressible components. The apparent viscosity and the yield stress are supposed to increase exponentially as temperature decreases. Section 4 is devoted to the heat equation. Finally, in Section 5 we prove the thermodynamical consistence of the model.

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## 2. Binary mixture of incompressible constituents

Lava is not a simple liquid but it is generally a mixture of silicate liquids, crystals, and gas bubbles. For sake of simplicity, we model lava as a mixture of only two incompressible fluids  $A$  and  $B$  whose intrinsic densities  $\rho_A, \rho_B$  are constant under standard conditions of temperature and pressure. We assume that  $\rho_A \neq \rho_B$ . We denote by  $\rho$  the density of lava.

Denoting by  $\mathbf{v}_A, \mathbf{v}_B$  the velocity of the fluids  $A$  and  $B$  respectively, the mean velocity  $\mathbf{v}$  is defined by

$$\rho \mathbf{v} = \rho_A \mathbf{v}_A + \rho_B \mathbf{v}_B.$$

We suppose that after the effusion the total amount of lava is contained into a fixed domain  $\Omega \subset \mathbb{R}^3$ . Let  $m$  be the total mass of lava in  $\Omega$ , i.e.

$$m = \int_{\Omega} \rho \, dv$$

and let  $m_A, m_B$  be the total masses of each component in  $\Omega$ . Accordingly,  $m = m_A + m_B$ . We introduce the apparent densities  $\tilde{\rho}_A, \tilde{\rho}_B$  of the two compounds  $A$  and  $B$  defined through the relations

$$m_A = \int_{\Omega} \tilde{\rho}_A \, dv, \quad m_B = \int_{\Omega} \tilde{\rho}_B \, dv.$$

Notice that the adjective ‘‘apparent’’ is used to emphasize that we are considering the integral over the total domain  $\Omega$ , rather than over the domain occupied by the constituent  $A$  or  $B$ . As a consequence,

$$\rho = \tilde{\rho}_A + \tilde{\rho}_B. \tag{1}$$

Accordingly, the ratio  $\tilde{\rho}_i/\rho_i, i = A, B$  denotes the volume fraction of the substance  $i$  and hence the following equality holds:

$$\frac{\tilde{\rho}_A}{\rho_A} + \frac{\tilde{\rho}_B}{\rho_B} = 1. \tag{2}$$

Once lava leaves the eruption vent and flows down the slopes of a volcano it suddenly quenches to a lower temperature and enters a level where it can no longer exist in equilibrium in its original homogeneous state. The two compounds begin to separate and form  $A$ -rich and  $B$ -rich fluid domains. During this phenomenon the total amount of each species in the whole domain must remain equal to the given original amount.

In contrast to two fluids approach, we consider the mixture as a single system obeying the laws of conservation of mass and linear momentum of continuum mechanics and we associate to each particle of the matter an additional scalar function  $\varphi$ , called order parameter, which varies continuously throughout  $\Omega$  and describes the phase of the material (see [12,11,13,14] and references therein). For instance, we define  $\varphi$  as the concentration difference between the two components of the mixture, that is

$$\varphi = \varphi_A - \varphi_B = \frac{\tilde{\rho}_A - \tilde{\rho}_B}{\rho},$$

where  $\varphi_i = \tilde{\rho}_i/\rho, i = A, B$  denotes the mass concentration of the fluid  $i$ . The equality

$$\varphi_A + \varphi_B = \frac{\tilde{\rho}_A + \tilde{\rho}_B}{\rho} = 1$$

leads to

$$\varphi_A = \frac{1 + \varphi}{2}, \quad \varphi_B = \frac{1 - \varphi}{2}. \tag{3}$$

It is apparent from equality (1) that  $\varphi \in [-1, 1]$ . In particular,  $\varphi = 1$  wherever only the component  $A$  occurs and  $\varphi = -1$  in regions where only the fluid  $B$  appears.

Furthermore, the definition of  $\varphi$  guarantees that the concentration difference of the two components is conserved in  $\Omega$  as the system evolves. Indeed, recalling the definition of  $\tilde{\rho}_A$  and  $\tilde{\rho}_B$ , we have

$$\int_{\Omega} \rho \varphi \, dv = \int_{\Omega} (\tilde{\rho}_A - \tilde{\rho}_B) \, dv = m_A - m_B = \text{constant}.$$

Unlike the two fluids model, in the diffusive approach the fundamental fields of the model are  $\rho, \mathbf{v}, \varphi$ , rather than  $\rho_A, \rho_B, \mathbf{v}_A, \mathbf{v}_B$  (see [15] for instance).

In our paper, we assume that the mixture, which is compounded by two incompressible fluids, is compressible, namely the density  $\rho$  may be not constant and may change owing to variations in the concentration parameter  $\varphi$ . We refer to the mixture as a quasi-incompressible fluid. For this reason,  $\rho$  is no more an independent variable, but it is a function of  $\varphi$ . In particular, (2) and (3) imply

$$\frac{1}{\rho} = \frac{1 + \varphi}{2} \cdot \frac{1}{\rho_A} + \frac{1 - \varphi}{2} \cdot \frac{1}{\rho_B},$$

which implies

$$\rho = \frac{2\rho_A\rho_B}{(\rho_A + \rho_B) - \varphi(\rho_A - \rho_B)}. \tag{4}$$

The assumption  $\rho_A \neq \rho_B$  assures that the density is not constant.

In order to guarantee the conservation of the total concentration over the whole domain, following [5,10] we postulate that  $\varphi$  obeys the diffusion equation

$$\rho \dot{\varphi} = \nabla \cdot \mathbf{J}, \tag{5}$$

where  $\mathbf{J}$  satisfies the boundary condition

$$\mathbf{J} \cdot \mathbf{n}|_{\partial\Omega} = 0. \tag{6}$$

In view of the transport and divergence theorems, we have the following equalities:

$$\frac{d}{dt} \int_{\Omega} \rho \varphi \, dv = \int_{\Omega} \rho \dot{\varphi} \, dv = \int_{\partial\Omega} \mathbf{J} \cdot \mathbf{n} \, da = 0.$$

As customary, we assume that the flux  $\mathbf{J}$  satisfies the Cahn-Hilliard law

$$\mathbf{J} = M(\varphi) \nabla \mu,$$

where  $M(\varphi)$  denotes the mass diffusivity which is supposed to be positive and  $\mu$  is the chemical potential. In particular, the boundary condition (6) is equivalent to

$$\nabla \mu \cdot \mathbf{n}|_{\partial\Omega} = 0.$$

The phenomenon of phase separation may be induced both by variations in the temperature and in the velocity. In addition, since the density of the mixture is not constant, the chemical potential depends explicitly on the pressure  $p$ . Thus, we assume that

$$\mu = -\frac{\gamma}{\rho} \nabla \cdot (\rho \nabla \varphi) - \frac{p}{\rho^2} \partial_{\varphi} \rho + \theta_0 F'(\varphi) + (\theta + \lambda \mathbf{v}^2) G'(\varphi), \tag{7}$$

where  $\gamma, \theta_0$  and  $\lambda$  are positive constants and  $F, G$  are suitable potentials characterizing the transition. In particular, we let

$$F(\varphi) = \frac{(\varphi - \varphi_0)^4}{4} - \frac{(\varphi - \varphi_0)^2}{2}, \quad G(\varphi) = \frac{(\varphi - \varphi_0)^2}{2},$$

where

$$\varphi_0 = \frac{m_A - m_B}{m_A + m_B}.$$

It is worth noting that if  $\varphi = \varphi_0$ , then the ratio  $\tilde{\rho}_A/\tilde{\rho}_B$  coincides with the ratio of the masses of the two fluids  $m_A/m_B$ .

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