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Vorticity and particle transport in periodic flow leaving a channel

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HIGHLIGHTS

- Emergence of instabilities and breaking of symmetry is dependent on Reynolds number.
- Dipoles can persist over more than one cycle and they can interact with each other.
- An asymptotic behavior appears as the Reynolds number increases.
- The integration of equations in 2D permits to recover some observational data.

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ABSTRACT

We investigate herein a periodically driven flow from a channel into an open domain. For this purpose, the equations of motion are solved with a pseudo spectral code based on a Chebyshev polynomial for the spatial coordinates and on a second-order finite difference method for time. During each driving period, the fluid that leaves the channel forms a coherent structure consisting of a pair of counter-rotating vortices, also known as a dipole. Dipole features, such as speed, intensity, and stability, depend on two dimensionless parameters: the Strouhal number and the Reynolds number. In some cases the dipole lifetime is greater than the driving period, so vortices may interact and even coalesce. The second part of the paper is devoted to calculating solid-particle trajectories immersed in this flow. For this purpose an equation deduced from first principles is solved considering drag, added mass, and history forces. We find that solid particles accumulate in certain regions and that a fraction of the particles leave the integration domain.

1. Introduction

We study a system consisting of a time-dependent flow through a domain formed by a channel connected to a basin. More precisely, we introduce a sinusoidal flow rate to mimic tidal-induced flow. When the fluid flushes from the channel into the basin, a pair of counter-rotating vortices is formed (known as a dipole). These systems can be observed in nature at different scales such as the mouths of rivers flushing into lakes or seas, or in channels connecting marine estuaries or bays with the sea [1].

Analytical, experimental, and observational data are available for this type of flow. For example, according to an inviscid model [2], hereafter referred to as the W-H model, if the Strouhal number $S < 0.13$ the dipole escapes the channel because the velocity attained after half of the driving period is such that the action of flow reversal cannot stop the dipole's self-propagation. For $S = 0.13$ the dipole remains stationary due to a balance between self-propagation and suction by the channel. Finally, for $S > 0.13$ a vortex pair forms and is entrained back toward the channel. In the aforementioned work the dipole is modeled by two counter-rotating vortex filaments. Therefore, the dipole moves with a self-induced velocity of

$$u = \frac{\Gamma}{2\pi d} \tag{1}$$

where Γ is the circulation and d is the distance between vortices [3].

A more realistic model for the dipole is the Lamb–Chaplygin vortex pair [4–6], which is an exact solution of the Euler equation. In this model vorticity is confined within a circle of radius R_0 inside of which are two finite-sized vortices. Numerical and experimental evidence indicates that the Lamb–Chaplygin dipole is a good approximation for moderate Reynolds number.

Nicolau del Roure et al. [7] and Bryant et al. [8] performed shallow-water laboratory experiments for tidal vortices in a tank and for four different inlet cases: a barrier island without channel, short and long jetties, and a barrier island with channel. In these experiments Nicolau del Roure et al. [7] measured the position of

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vortices, the maximal vorticity, the circulation, and the effective diameter. They used dye visualization to make their measurements and the velocity at the free surface was obtained using particle image velocimetry (PIV). Their investigation covers a broad range of Strouhal numbers. For $S = 0.13$ they find that the dipole effectively attains a stationary position, but the distance to the channel is different from that predicted by the W-H model.

Amoroso & Gagliardini [9] studied hydrographic processes in the San Jose and San Matias Gulfs in Patagonia. Both gulfs are connected by a narrow channel. They used observational data provided by the satellite systems of Landsat and the National Oceanographic and Atmospheric Administration (NOAA). The maps of free surface circulation reveal the existence of vortices with lifetimes greater than one tidal cycle; furthermore, trains of vortices are observed.

The vortices that escape from the channel mouth have a finite lifetime. They evolve for a while and are finally destroyed. Nicolau del Roure et al. [7] associate this process to bottom friction, but other mechanisms are also involved. For example, [10] analyzed the stability and found a sinusoidal symmetric instability in the long-wavelength range. More recently, [11] investigated the linear stability of the Lamb–Chaplygin dipole with respect to three-dimensional perturbations for Reynolds numbers of 400 and 10,000. They found that the most unstable mode is asymmetric in the short-wavelength range. Furthermore, [12] experimentally investigated the stability of a pair of vortices produced by the rotation of a two flat plates. They focused on modes with wavelengths ranging from the size of the core vortex to the size of the intervortex spacing. They found that the instability deforms the core, which agrees well with predictions of the theory of elliptic instability. In addition, they found that an array of secondary vortices leads to the destruction of the dipole.

In this paper, we remain within the shallow-water approximation because the characteristic size of the dipole is sufficiently greater than the depth of the fluid layer. Usually it is adequate to consider that the flow properties depend weakly on the vertical coordinate, except in a thin layer near the bottom. However, evidence exists that contradicts this hypothesis. For example, [13] produced a dipole by a turbulent impulsive jet in a shallow-water experiment with Reynolds number between 50,000 and 75,000. Besides a pair of counter-rotating vortices, they report the emergence of vertical motion ahead of the dipole. Lacaze et al. [14], Albagnac [15] and Albagnac et al. [16] reported a similar behavior in a laminar flow; they produced a dipole by rotating two vertical plates in a rectangular basin. They also observed vertical motion; in fact a spanwise vortex was detected in front of the dipole.

In the same sense, [17] numerically simulated a dipole in a thin horizontal layer. As initial conditions, they proposed a velocity field such as the Lamb–Chaplygin vortex pair in the horizontal plane and a vertical Poiseuille velocity profile. The two relevant parameters are the Reynolds number and the aspect ratio $\delta = H/R_0$, where H is the fluid-layer depth and R_0 is the radius of the Lamb–Chaplygin vortex. They found that the three-dimensional nature of the flow depends on the single parameter $K = \delta^2 Re$. For $K < 6$ the flow is dominated by the viscosity, so the vertical motion can be neglected. In the range $6 < K < 15$, the dipole properties are modified by the vertical motion and a spanwise vortex appears in front of the dipole. Finally, for $K > 15$ the three-dimensional nature of the flow is well developed and the intensity of the spanwise vortex is comparable with that of vortices in the dipole.

The other aspect we treat in this work is the transport of particles by the flow. This subject has been investigated extensively in the recent past. Several studies addressed the transport of particles in flow with vortices. For example, [18] experimentally determined the transport during the formation and growth of an annular vortex. The ring vortex was produced with a piston–cylinder apparatus immersed in a water tank. The dynamical and geometrical characteristics were deduced from measurements of the velocity field in a plane passing through the axis of symmetry. They found that, in the early stage, most of the fluid that enters the region of nonzero vorticity comes from the cylinder. As the vortex ring grows and moves, fluid outside this cylinder is entrained.

Angilella [19] studied the transport of dust in the vicinity of a pair of identical point vortices. In this case vortices rotate about a common center and remain in a vertical plane. The research was motivated by the fact that a pair of corotating vortices increases the particle dispersion. The forces considered in the analysis were gravity, drag, and the Coriolis and centrifugal (pseudo) forces; the latter two were introduced because the equation of motion was solved in a rotating frame of reference. When drag is the dominant force, the particle trajectories exhibit chaotic behavior, so mixing is enhanced.

On the other hand, under certain conditions the paths of small solid particles are not appreciably different from that of the fluid elements. This property is used in PIV, where the velocity of a flow is calculated from the displacement of particles in two successive frames. In water measurements, flow is seeded with particles with size typically in the range of 10–50 μm in diameter. In addition, for the density to be similar to that of the surrounding fluid, hollow glass or polyamide spheres are often used.

The aim of this work is to study the evolution of dipoles for different values of Strouhal and Reynolds numbers. Particular attention is devoted to the case when the dipole lifetime exceeds the driving period, so vortices interact. In addition, we investigate the stability of the flow responsible for breaking the symmetry and destroying the vortices. On the other hand, we are interested in calculating solid–particle paths from an equation deduced from first principles [20], and from this to determine if particles accumulate in some particular region. Finally, a goal of this work is to compare our numerical results with experimental and observational data. This comparison shows that this two-dimensional numerical simulation reproduces some important features of tidal-induced flow.

The paper is organized as follows: In Section 2, we present the differential equations in stream function–vorticity formulation and describe the geometry of system and the boundary conditions imposed. In Section 3, we describe the numerical solution based on Chebyshev polynomials for spatial coordinates and finite differences for time. In Section 4, we present the evolution of the vorticity and dipole position for different values of S and Re. In Section 5, we solve the equation for solid–particle trajectories and show some particular cases. Section 6 is devoted to a discussion of the main results and, finally, we draw conclusions in Section 7.

2. Theoretical framework and methodology

The Navier–Stokes and continuity equations express the main conditions satisfied by moving fluids. For incompressible flow the equations are:

$$\frac{D\vec{u}}{Dt} = \vec{F} - \frac{1}{\rho}\vec{\nabla}P + \nu\nabla^2\vec{u} \tag{2}$$

$$\vec{\nabla}\cdot\vec{u} = 0 \tag{3}$$

where

$$\frac{D\vec{u}}{Dt} = \frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u} \tag{4}$$

is the material derivative, ν is the kinematic viscosity, and P is the pressure. Because the solution is obtained in two dimensions (2D), we use the stream function–vorticity formulation [21]. In this manner only two second-order partial differential equations need to be solved.