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Heat transport and flow structure in rotating Rayleigh–Bénard convection

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ABSTRACT

Here we summarize the results from our direct numerical simulations (DNS) and experimental measurements on rotating Rayleigh–Bénard (RB) convection. Our experiments and simulations are performed in cylindrical samples with an aspect ratio Γ varying from 1/2 to 2. Here $\Gamma = D/L$, where D and L are the diameter and height of the sample, respectively. When the rotation rate is increased, while a fixed temperature difference between the hot bottom and cold top plate is maintained, a sharp increase in the heat transfer is observed before the heat transfer drops drastically at stronger rotation rates. Here we focus on the question of how the heat transfer enhancement with respect to the non-rotating case depends on the Rayleigh number Ra, the Prandtl number Pr, and the rotation rate, indicated by the Rossby number Ro. Special attention will be given to the influence of the aspect ratio on the rotation rate that is required to get heat transport enhancement. In addition, we will discuss the relation between the heat transfer and the large scale flow structures that are formed in the different regimes of rotating RB convection and how the different regimes can be identified in experiments and simulations.

1. Introduction

Rayleigh–Bénard (RB) convection, i.e. the flow of a fluid heated from below and cooled from above, is the classical system to study thermally driven turbulence in confined space [1,2]. Buoyancy-driven flows play a role in many natural phenomena and technological applications. In many cases the fluid flow is also affected by rotation, for example, in geophysical flows, astrophysical flows, and flows in technology [3]. On Earth, many large-scale fluid motions are driven by temperature-induced buoyancy, while the length scales of these phenomena are large enough to be influenced by the Earth's rotation. Key examples include the convection in the atmosphere [4] and oceans [5], including the global thermohaline circulation [6]. These natural phenomena are crucial for the Earth's climate. Rotating thermal convection also plays a significant role in the spontaneous reversals of the Earth's magnetic field [7]. Rotating RB convection is the relevant model to study the fundamental influence of rotation on thermal convection in order to better understand the basic physics of these problems.

In this paper we discuss the recent progress that has been made in the field of rotating RB convection. First we discuss the dimensionless parameters that are used to describe the system. Subsequently, we give an overview of the parameter regimes in which the heat transport in rotating RB is measured in experiments and direct numerical simulations (DNS). This will be followed by a description of the characteristics of the Nusselt number measurements and a description of the flow structures in the different regimes of rotating RB. Finally, we address how the different turbulent states are identified in experiments and simulations by flow visualization, detection of vortices, and from sidewall temperature measurements.

2. Rotating RB convection

When a classical RB sample is rotated around its center axis, it is called rotating RB convection. For not too large temperature gradients, this system can be described with the Boussinesq approximation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} + 2\boldsymbol{\Omega}\times\mathbf{u} = -\nabla p + \nu\nabla^2\mathbf{u} + \beta g\theta\hat{z}, \quad (1)$$

$$\frac{\partial \theta}{\partial t} + \mathbf{u}\cdot\nabla\theta = \kappa\nabla^2\theta, \quad (2)$$

for the velocity field $\mathbf{u}$, the kinematic pressure field p, and the temperature field θ relative to some reference temperature. In the Boussinesq approximation it is assumed that the material properties of the fluid such as the thermal expansion coefficient

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β, the viscosity ν, and the thermal diffusivity κ do not depend on temperature. Here g is the gravitational acceleration and $\boldsymbol{\Omega}$ is the rotation rate of the system around the center axis, pointing against gravity, $\boldsymbol{\Omega} = \Omega\hat{\mathbf{z}}$. Note that $\rho(T_0)$ has been absorbed in the pressure term. In addition to the Oberbeck–Boussinesq equations we have the continuity equation ($\nabla \cdot \mathbf{u} = 0$). A no-slip ($\mathbf{u} = 0$) velocity boundary condition is assumed at all the walls. Moreover, the hot bottom and the cold top plate are at a constant temperature, and no lateral heat flow is allowed at the sidewalls.

Within the Oberbeck–Boussinesq approximation and for a given cell geometry, the dynamics of the system is determined by three dimensionless control parameters, namely, the Rayleigh number

$$Ra = \frac{\beta g \Delta L^3}{\kappa \nu}, \tag{3}$$

where L is the height of the sample, $\Delta = T_b - T_t$ the difference between the imposed temperatures T_b and T_t at the bottom and the top of the sample, respectively, the Prandtl number

$$Pr = \frac{\nu}{\kappa}, \tag{4}$$

and the rotation rate which is indicated by the Rossby number

$$Ro = \sqrt{\beta g \Delta / L}/(2\Omega). \tag{5}$$

The Rossby number indicates the ratio between the buoyancy and Coriolis force. Note that the Ro number is an inverse rotation rate. Alternative parameters to indicate the rotation rate of the system are the Taylor number

$$Ta = \left(\frac{2\Omega L^2}{\nu}\right)^2, \tag{6}$$

comparing Coriolis and viscous forces, or the Ekman number

$$Ek = \frac{\nu}{\Omega L^2} = \frac{2}{\sqrt{Ta}}. \tag{7}$$

A convenient relationship between the different dimensionless rotation rates is $Ro = \sqrt{Ra/(PrTa)}$.

The cell geometry is described by its shape and an aspect ratio

$$\Gamma = D/L, \tag{8}$$

where D is the cell diameter. The response of the system is given by the non-dimensional heat flux, i.e. the Nusselt number

$$Nu = \frac{QL}{\lambda \Delta}, \tag{9}$$

where Q is the heat–current density and λ the thermal conductivity of the fluid, and a Reynolds number

$$Re = \frac{UL}{\nu}. \tag{10}$$

There are various reasonable possibilities to choose a velocity U, e.g., the components or the magnitude of the velocity field at different positions, local or averaged amplitudes, etc., and several choices have been made by different authors. A summary of some work that has been done on rotating RB convection is given in Section 2.8 of the book by Lappa [8].

3. Parameter regimes covered

In Fig. 1 we present the explored Ra–Pr–Ro parameter space for rotating RB convection.[2] Here we emphasize that numerical simulations and experiments on rotating RB convection are complementary, because different aspects of the problem can be addressed. Namely, in accurate experimental measurements of the heat transfer a completely insulated system is needed. Therefore, one cannot visualize the flow while the heat transfer is measured. On the positive side, in experiments one can obtain very high Ra numbers and long time averaging. In simulations, on the other hand, one can simultaneously measure the heat transfer while the complete flow field is available for analysis. But the Ra number that can be obtained in simulations is lower than in experiments, due to the computational power that is needed to fully resolve the flow. Here we should also mention that the highest Ra number reached in rotating RB experiments is almost $Ra = 10^{16}$, whereas in DNS of rotating RB convection the highest Ra number is $Ra = 4.52 \times 10^9$. However, the flexibility of simulations allows to study more Pr numbers, i.e. covering a range of $0.2 \le Pr \le 100$, whereas the present experiments are almost exclusively for $3.05 \le Pr \le 6.4$, i.e. the Pr number regime accessible with water. In addition, we note that the $1/Ro$ number regime that can be covered in experiments can, depending on Ra and Pr, be somewhat limited. Very low $1/Ro$ values, corresponding to very weak rotation, are difficult due to the accuracy limitations at very small rotation rates. The lowest rotation rates achieved in the recent Eindhoven [10,11,9] and Santa Barbara [12,13] experiments are about 0.01 rad/s (one rotation every 10.5 min). For the Eindhoven experiments this is already rather close to the accuracy with which we can set the rotation rate. The highest $1/Ro$ that can be obtained in a setup is either determined by the requirement that the Froude number $Fr = \Omega^2(L/2)/g$ [14], which indicates the importance of centrifugal effects, is not too high (usually $Fr < 0.05$ is considered to be a safe threshold [15]) or by the maximum rotation rate that can be achieved.

4. Nusselt number measurements

Early linear stability analysis, see e.g. Chandrasekhar [30], revealed that rotation has a stabilizing effect due to which the onset of heat transfer is delayed. This can be understood from the thermal wind balance, which implies that convective heat transport parallel to the rotation axis is suppressed. Experimental and numerical investigations concerning the onset of convective heat transfer and the pattern formation in cylindrical cells just above the onset under the influence of rotation have been performed by many authors, see e.g. [31–45].

Since the experiments by Rossby in 1969 [46], it is known that rotation can also enhance the heat transport. Rossby found that, when water is used as the convective fluid, the heat transport first increases when the rotation rate is increased. He measured an increase of about 10%. This increase is counterintuitive as the stability analysis of Chandrasekhar [30] has shown that rotation delays the onset to convection and from this one would expect that the heat transport decreases. The mechanism responsible for this heat transport enhancement is Ekman pumping [46–48,26,23,15], i.e. due to the rotation, rising or falling plumes of hot or cold fluid are stretched into vertically-aligned-vortices that suck fluid out of the thermal boundary layers adjacent to the bottom and top plates. This process contributes to the vertical heat flux. For stronger rotation Rossby found, as expected, a strong

[2] In Fig. 1 of Stevens et al. [9] also the Ra–Ro–Γ parameter space for $Pr = 4.38$ can be found.