European Journal of Mechanics B/Fluids 31 (2012) 53–70

Contents lists available at SciVerse ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Double-deck structure revisited

R. Yapalparvi *

Department of Mathematics, 1280 Main St W, McMaster University, Hamilton, Canada

A R T I C L E   I N F O

Article history:
Received 13 January 2011
Received in revised form
26 April 2011
Accepted 26 May 2011
Available online 6 June 2011

Keywords:
Double-deck structure
Separation
Jet-like boundary layer
Corners
Humps/indentations
Finite-difference–Chebyshev collocation technique

A B S T R A C T

We investigate a problem based on the double-deck structure of Smith & Duck (1977) [1] numerically. In the first part of the paper, we discuss the nature of separation and subsequent reversed flow occurring when a jet-like boundary layer encounters a corner with angle $\theta \propto Re^{-\frac{3}{14}}\alpha$, where α is an order-one quantity and Re is the Reynolds number, which is assumed to be large. The separation is caused by a nonlinear upstream response within the jet, wherein the motion acquires a double-deck structure. We also consider the behaviour of a jet-like boundary layer when it encounters a hump or an indentation. The boundary shape was chosen to be $y = h\exp(-k^2x^2)$ for a range of h and k for both humps ($h > 0$) and indentations ($h < 0$), where h is the height of the hump/indentation and k is the width parameter. For higher values of h in the case of humps we observe a recirculation region ahead and behind the hump. In the case of an indentation, the recirculation region is centred at the indentation.

In the second part of the paper, we numerically investigate the steady flow of a liquid layer past obstacles at high Reynolds number ($Re \to \infty$). We discuss the nature of separation and subsequent reversed flow occurring when a liquid layer encounters a convex corner. The angle is represented as $\theta \propto Re^{-\frac{3}{7}}\alpha$, where α is an order-one quantity and Re is the Reynolds number, which is assumed to be large. We again consider the behaviour of a liquid layer when it encounters a hump or indentation. The boundary shape was chosen to be same as for the wall-jet case. The pressure–displacement law is a combination of a special case of that occurring in the hypersonic flow theory of Brown et al. (1975) [25] and the jet law of Smith & Duck (1977) [1] given by $p = -A' - \sigma A$, where σ is inversely proportional to the angle of inclination of the initial plane. The pressure distribution for values $\sigma \geq 1$ suggests that there is no longer any local minimum or maximum, but only a favourable pressure gradient, suggesting no separation. For liquid layer flows past a hump, free interaction takes place, together with a hydraulic jump occurring far ahead of the hump. For higher values of $\sigma \geq 2$, we found oscillatory behaviour in pressure and skin friction distributions which decays far downstream for both humps and indentations.

We use a numerical technique based on a finite-difference technique in the streamwise direction and Chebyshev collocation in the normal direction. The resulting algebraic equations are linearised using the Newton–Raphson technique which leads to a block pentadiagonal system which is solved using a direct method.

1. Introduction

The motivation for the present work is to understand small and moderate scale separation when wall jets encounter corners and humps/indentations. This problem is based on the double-deck structure of Smith & Duck [1] and is investigated numerically. This problem was studied by Merkin & Smith [2] who were able to obtain solutions up to an angle $\alpha = 10$. Our contribution is to compute solutions for the corner problem with the scaled angle parameter larger than those computed before, show an existence of a secondary separation region embedded within a large primary separation region and also obtain the critical angle for the onset of separation for the case of corners. We resolve a disparity in the "plateau" value of pressure between the two values given by Smith & Duck [1] and Merkin & Smith [2]. Merkin & Smith restrict their outer boundary to $y_\infty = 14$ in their computations. We show that restricting the outer boundary to such a value would lead to "overshooting" of skin friction. Merkin [3] also studied wall jets encountering humps/indents. We obtain solutions with large values of $|h|$ and varying width parameter k, where h and k are the height and width of the hump/indent. We show the presence of a separated region upstream of the hump, which was not previously computed by Merkin [3].

The same double-deck structure was used by Gajjar [4] to study liquid layer flow past convex corners. We show that there is no separation for $\sigma \geq 1$, which suggests that there is no longer

* Tel.: +1 289 339 9908.
E-mail address: ramesh.yapalparvi@gmail.com.

0997-7546/$ – see front matter © 2011 Elsevier Masson SAS. All rights reserved.
doi:10.1016/j.euromechflu.2011.05.007

any local pressure minimum or maximum, but only a favourable pressure gradient. We also investigate the case of a liquid layer encountering a concave shaped obstacle such as a hump. We substantiate the evidence that when a liquid layer encounters a hump, i.e., when $p > 0$, free interaction takes place which results in a hydraulic jump occurring far ahead of the obstacle.

Flow separation is an interesting phenomenon associated with fluid flows at large Reynolds number. Of particular interest is the aerodynamic situation, where a rigid body is placed in a uniform stream. Recall, that the Reynolds number is the dimensionless ratio of inertial to viscous forces. High Reynolds number flows correspond to relatively fast flows of fluids with relatively small viscosity, for example "common" gases and liquids, such as air and water. Boundary-layer theory has provided a framework for investigating many aspects of fluid flow at high Reynolds numbers. In 1904, Prandtl [5] observed that despite "common" gas and liquids having low viscosity, viscous effects did play a major role in the separation phenomenon. He argued that high Reynolds number flow around a rigid body may be treated as inviscid everywhere except in a very thin region adjacent to the body surface, the so-called boundary layer. Within this region, which has a thickness of $O(Re^{-1/2})$, viscosity reduces the tangential velocity u from the slip velocity predicted by the inviscid theory to zero at the surface. However, under the influence of adverse mainstream pressure gradient, the low momentum fluid adjacent to the wall is susceptible to the onset of reversed flow, which leads to boundary-layer separation and an interaction with the outer inviscid flow.

Prandtl's description of the separation process, revealing that it was at the time, still left some important questions unanswered. In particular, it remained unclear why the recirculation flow region did not remain inside the boundary layer whose thickness decreases with the Reynolds number as $O(Re^{-1/2})$. Instead, experiments show that eddies erupt from the boundary layer, resulting in the formation of large recirculation regions which influence the entire flow field around the body. This question was answered in the late 1930s when it was established that the solution to the boundary-layer equations (in their classical formulation as given by Prandtl) leads to a singularity at the separation point; see [6,7]. The form of the singularity was first predicted by Landau & Lifshitz [8]. Making use of heuristic arguments, they arrived at the conclusion that the skin friction, τ_w, decreases on approach to the separation point as

$$\tau_w \sim \sqrt{x_s - x}.$$

Here x is the distance along the body surface, and x_s is the position of the separation point. They also found that the velocity component normal to the body surface experiences unbounded growth inversely proportional to $\sqrt{x_s - x}$. Later, Goldstein [9] confirmed this result, and (even more important) proved that the singularity at the separation precludes the continuation of the solution beyond the point of zero skin friction. This clearly shows that the boundary-layer theory in its classical form, as formulated by Prandtl [5], cannot be used in a small vicinity of the separation point. A key element of the separation process, which was not fully appreciated in Prandtl's description, was an interaction between the boundary layer and external inviscid flow, now referred to as the viscous–inviscid interaction. Asymptotic theory of the viscous–inviscid interaction, also known as the triple-deck theory, was formulated simultaneously by Neiland [10] and Stewartson & Williams [11] for the self-induced separation in supersonic flow and by Stewartson [12] and Messiter [13] for incompressible fluid flow near the trailing edge of a flat plate. Later, many researchers were involved in the development of the theory, and it became clear that the viscous–inviscid interaction plays a key role in a wide variety of fluid dynamic phenomena. An exposition of applications of the theory to different forms of the boundary-layer separation may be found, for example, in the monograph by Sychev et al. [14], review papers by Stewartson [15], Smith [16], also in the book by Sobey [17] and recently by Lagrée [18].

An important application of viscous–inviscid interaction theory is the analysis of the flow separation observed in near-wall jets and thin films. The need to investigate the structure of a jet flow near its point of separation from a wall arises in many situations, for example flows in pipes/channels, oscillatory motions and thermal or colliding boundary layers, as well as wall jets near corners and other wall discontinuity conditions. The interaction might be a result of self-induced separation, a corner point or a trailing edge. We study whether the ideas from the triple-deck theory can be extended to the situation when there is no outer flow wherein the boundary layer is driven by internal (buoyancy) forces, and the flow in the boundary layer exhibits a jet-like profile. The answer to the above question was given by Smith & Duck [1], who studied the problem of separation of jets or thermal boundary layers from a wall. They showed that jet flow can develop free interactions which have a double-deck structure in which the unknown induced pressure is due to the centrifugal forces acting across the jet. Smith & Duck [1] then applied these results to understand the behaviour of flows at a corner and the collision of two oncoming jets. They also showed that during interaction the fluid near the wall forms a viscous sublayer, driven along by the induced local pressure gradient, whereas majority of the boundary layer reacts in an inviscid displaced fashion. Upstream of the separation, the sublayer pressure increases slightly, causing a decrease in the skin friction, and the sublayer expands. The associated movement of fluid in the inviscid region then induces a pressure drop across the jet, but, because the pressure at the edge of the jet does not change, the transverse pressure gradient reinforces the pressure increase of the wall. They concluded that separation, followed by a sizeable eddy of reversed flow, takes place over a streamwise length scale of distance $O(Re^{-\frac{3}{7}})$ along the wall. The scalings involved were mainly derived from an order-of-magnitude argument analogous to that used by Smith [19] in a channel flow problem.

Messiter & Liñan [20] studied the behaviour of a free convection boundary layer on a vertical plate near a discontinuity in plate temperature and near the trailing edge of a vertical plate. Smith [21] also studied the problem of free convection boundary layer encountering a trailing edge. Messiter & Liñan [20] and Smith [21] both showed that the boundary layer adjusts to the discontinuity in boundary conditions through a "double-deck" structure. Later Merkin & Smith [2] applied this theory to free convection boundary layers near the corner of a body contour and at the trailing edge of a flat plate. They found that the corner problem has some similarities with supersonic flow near a convex corner, as discussed by Stewartson [15], though the pressure–displacement relation is different. They concluded that, for concave corners with sufficiently large angles, there will be a reversed-flow region centred on the corner and that for convex corners the flow will separate downstream of the corner. Merkin & Smith [2] were able to compute the solution for the concave corner up to $\alpha = 10$ and reported a "plateau" value of 1.6 which was different from the one reported by Smith & Duck [1]. This disparity in the "plateau" value motivated us to investigate this problem as explained earlier.

In this paper, we shall consider the behaviour of a jet-like boundary layer when it encounters a small hump or an indentation. We study a wider range of parameter values for the problems studied by Merkin [3]. The humps and indentations considered are of the form $\hat{y} = Re^{-\frac{9}{14}} h \exp(-k^2X^2)$ and $\hat{x} = Re^{-\frac{3}{8}}X$. Here h and k are constants corresponding to the height and spread of the hump or indentation, respectively, in the lower deck scalings. For small transverse humps and indentations, i.e., $|h| \ll 1$ and $k = O(1)$, Merkin [3] obtained both analytical solutions for