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Comparative study on the accuracy and stability of SPH schemes in simulating energetic free-surface flows

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ABSTRACT

Free-surface flows are of significant interest in Computational Fluid Dynamics (CFD). However, modelling them especially when the free-surface breaks or impacts on solid walls can be challenging for many CFD techniques. Smoothed Particle Hydrodynamics (SPH) has been reported as a robust and stable method when applied to these problems. In modelling incompressible flows using the SPH method an equation of state with a large sound speed is typically used. This weakly compressible approach (WCSPH) results in a stiff set of equations with a noisy pressure field and stability issues at high Reynolds number. As a remedy, the incompressible SPH (ISPH) technique was introduced, which uses a pressure projection technique to model incompressibility. Although the pressure field calculated by ISPH is smooth, the stability of the technique is still an open discussion. An alternative approach is to use an acoustic Riemann solver and replace the particle velocities and pressures by pressures and velocities determined from a Riemann solver. This technique is equivalent to the Godunov method in Eulerian techniques and so will be called the Godunov SPH method (GSPH). However, since the acoustic Riemann solver is a first order approximation of the Riemann solution, it is highly dissipative and cannot be employed in energetic free-surface flows without modification. In this paper, the GSPH method is modified by using the HLLC (Harten Lax and van Leer-Contact) Riemann solver. The accuracy of the modified GSPH technique is further improved by utilising the MUSCL (Monotone Upstream-centred Schemes for Conservation Laws) scheme with Slope–Limiter. This modified GSPH method along with the WCSPH and ISPH techniques are used to study non-linear sloshing flow. The accuracy, stability and efficiency of the techniques are assessed and the results are compared with experimental data.

1. Introduction

The Smoothed Particle Hydrodynamics (SPH) method is a meshless, purely Lagrangian technique which was originally developed in 1977 by Lucy [1] and Monaghan and Gingold [2,3]. It has subsequently been successfully employed in a wide range of problems, e.g. astrophysics [4–6], fluid mechanics [7,8], solid mechanics [9–11], fluid–structure interaction [12–14] and many more (see [15] for a recent review).

In SPH, the "particles" are moving nodes that are advected with the local velocity and carry field variables such as pressure and density. As the fields are only defined at the set of discrete points, smoothing (interpolation) kernels are used to define a continuous field and to ensure differentiability. Incompressibility is typically satisfied in one of two ways. In the most common approach, termed weakly compressible SPH (WCSPH), the fluid is assumed compressible with a large sound speed (such that the Mach number $M \approx 0.1$ and the density of the fluid typically varies by less than 1%). An alternative approach uses a fractional-step projection technique. In the first step, the velocity is integrated in time without enforcing incompressibility. Incompressibility is then achieved by projecting the intermediate velocity field onto a divergent-free space by solving a pressure Poisson equation [16]. This approach will be referred to as incompressible SPH (ISPH).

While the WCSPH scheme has been successfully implemented at low Reynolds number, the reflection of sound waves off boundaries at high Reynolds numbers lead to severe instabilities in the scheme [17]. In addition the stiff equation of state can result in large unphysical pressure fluctuations which also affects stability. These fluctuations in the pressure field can be mitigated by reducing the sound speed and thereby relaxing the system.

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Increasing the viscosity can also reduce the noise in the pressure field as an increased viscosity leads to smoother velocities and therefore smoother densities in the equation of state. However both of these approaches typically compromise accuracy and are therefore not solutions to the noisy pressure problem.

As a remedy for large unphysical pressure oscillations, Colagrossi and Landrini [18] corrected the density calculation by renormalizing the density using Moving-Least Squares (MLS) density correction. They showed that the correction improves the mass-area-density consistency and also filters out pressure oscillations. They also found that the density re-initialization procedure is beneficial with respect to energy conservation when it is used along with artificial viscosity. Others [19–21] have proposed a δ-SPH scheme by modifying the SPH equations and adding a proper artificial diffusive term into the continuity equation in order to remove the spurious numerical high-frequency oscillations in the pressure field. However, these WCSPH issues along with the requirement of a very small time-step in order to resolve the sound waves can be alleviated by using ISPH. Nevertheless, the solution of the elliptic pressure Poisson equation (PPE) increases the computational costs at each time-step. Cummins and Rudman [16] showed that the computational costs of solving the PPE can be reduced by use of a multi-grid technique.

Another weakness of the WCSPH method is the use of an artificial viscosity to introduce dissipation and therefore ensure stability. However, the dissipation can be achieved in a more accurate manner by solving the Riemann problem between each pair of particles. The concept and advantages of using Riemann Solvers in SPH has been studied by several researchers [22–25]. Monaghan [26] showed that the artificial viscosity is analogous to terms constructed from signal velocities and jumps in variables across characteristics in the Riemann problem. However, it was pointed out that the results with artificial viscosity were not as accurate as well approximated Riemann solutions [26]. To overcome this, Parshikov and Stanislav [27] and Parshikov et al. [23] proposed a modified SPH method using a first order approximation of the acoustic Riemann solver, which does not require an artificial viscosity term for dissipation. Inutsuka [24] has reformulated the SPH method using a second order Riemann solver. Cha and Whitworth [25] have derived four different formulations for Godunov-type particle hydrodynamics (GPH). They have performed a von Neumann stability analysis for GPH formulations and concluded that GPH is stable for all wavelengths, while SPH is unstable for certain wavelength. Molteni and Bilello [28] have presented two approaches for implementing Godunov schemes in SPH and tested their Godunov-type SPH formulations for a classical 1D shock tube problem and a 2D shock around a Black Hole. They observed a significant improvement in the accuracy of the SPH method. Roubtsova and Kahawita [29] demonstrated that the SPH method with the Riemann solver introduced by Parshikov and Stanislav [27] can be successfully applied to free-surface flows. Ferrari et al. [30] have proposed a modified SPH scheme based on introducing the monotone Rusanov flux scheme in the density equation and hence removing any artificial viscosity term. They showed that the modified SPH scheme is able to compute an accurate and little oscillatory pressure field. Leduc et al. [31] proposed a multi-fluid SPH formulation by using an acoustic Riemann solver. They applied the SPH Riemann solver proposed by Parshikov et al. [23] to several multi-fluid flows problems. It was observed that the results were affected by high numerical diffusion [32]. To alleviate this numerical diffusion linked with the proposed upwind schemes, they modified their Riemann solver by using a preconditioned Riemann solver [32]. Leduc et al. [32] obtained a new linearised Riemann solver for multi-fluid flows when surface tension effects are important. This is in contrast to this particular study which focuses on modelling free surface flows without surface tension effects. Recently, Fourey et al. [33,34] used Riemann solver to stabilise their SPH solver coupled with a Finite Element (FE) solver for studying fluid–structure interaction. They showed that using a Riemann solver resulted in a smoother SPH pressure field passed to the FEM solver. This consequently improved the accuracy of their coupled SPH-FEM solver [33,34]

In this paper, we apply the HLLC Riemann solver [35] to the WCSPH method [36,37]. The HLLC Riemann solver is the modified Riemann solver proposed by Harten et al. (HLL) [38] which considers the contact discontinuity wave in the Riemann problem. It was shown that the HLLC Riemann solver is as accurate and robust as the exact Riemann solver but computationally more efficient [39]. To reduce the dissipation and increase its spatial order, the MUSCL scheme with a slope limiter is employed in this work.

The purpose of this paper is to compare the accuracy and stability of the GSPH method against the traditional WCSPH and ISPH techniques for non-linear high Reynolds number sloshing flows. Here, the WCSPH and ISPH techniques are first described including the implementation of free-surface and solid wall boundary conditions. This is followed by an outline of the GSPH method with the HLLC Riemann solver. Similar to [36,37] an MUSCL-based slope limiter scheme was used to improve the spatial order of the Riemann solver. However, a more simpler and general form of the Godunov-type SPH governing equations based on the solution of a 1D local Riemann problem along the interaction line of a pair of interacting particles is proposed. This form of Godunov-type SPH can accurately simulate energetic flows at various Reynolds numbers and can be easily extended to three-dimensional (3D) problems without any modification. Finally, the accuracy, stability and efficiency of WCSPH, ISPH and GSPH are studied in the simulation of sloshing flow. We compare both accuracy and efficiency of the three methods to assess not only which method provides smoother, more accurate results but also whether such accuracy comes at a computational cost. If a computational cost is incurred, we quantify the cost as the resolution changes. The results are also compared with experimental data for both global features (free-surface profile) and local features (time variation of impact pressure on the boundaries).

2. The SPH method

The SPH method uses smoothing kernels to express a function in terms of its values at a set of disordered points. The smoothing kernel function (or weighting function), specifies the contribution of a typical field variable, $A(r)$, at position, r, in space. The kernel estimate of $A(r)$ is defined as [40,41]

$$A(\mathbf{r}) = \int_V A(\mathbf{r}')W(\mathbf{r}-\mathbf{r}', h)d\mathbf{r}' \tag{1}$$

where V represents the solution space and the smoothing length h represents the effective width of the kernel W. The kernel has the following properties

$$\int_V W(\mathbf{r}-\mathbf{r}', h)d\mathbf{r}' = 1, \qquad \lim_{h\to 0} W(\mathbf{r}-\mathbf{r}', h) = \delta(\mathbf{r}-\mathbf{r}'). \tag{2}$$

If $A(\mathbf{r}')$ is known only at a discrete set of N points $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N$, then we approximate $A(\mathbf{r}')$ as,

$$A(\mathbf{r}') = \sum_{j=1}^{N} \delta(\mathbf{r}'-\mathbf{r}_j)A(\mathbf{r}_j)(dV)_j \tag{3}$$

where the index j denotes the particle label and particle j has a mass m_j and density ρ_j at position $\mathbf{r}_j$. The differential volume element