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Applying laser Doppler anemometry inside a Taylor–Couette geometry using a ray-tracer to correct for curvature effects

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A B S T R A C T

In the present work it will be shown how the curvature of the outer cylinder affects laser Doppler anemometry measurements inside a Taylor–Couette apparatus. The measurement position and the measured velocity are altered by curved surfaces. Conventional methods for curvature correction are not applicable to our setup, and it will be shown how a ray-tracer can be used to solve this complication.

By using a ray-tracer the focal position can be calculated, and the velocity can be corrected. The results of the ray-tracer are verified by measuring an a priori known velocity field, and after applying refractive corrections good agreement with theoretical predictions are found. The methods described in this paper are applied to measure the azimuthal velocity profiles in high Reynolds number Taylor–Couette flow for the case of outer cylinder rotation.

1. Introduction

A Taylor–Couette (TC) apparatus consists of two coaxial, differentially rotating, cylinders, see Fig. 1. The annulus between the cylinders is filled with a working fluid; most commonly, as in our case, water is chosen. The apparatus has been used to study hydrodynamic instabilities, pattern formation, turbulence, and was found to have a rich phase diagram with different types of flow structures [1–11]. To get a deep understanding of these phenomena it is crucial to measure the local flow velocity.

Measuring the velocity field inside a TC apparatus was done for a long time using intrusive measurement techniques, *e.g.* constant temperature anemometry [12–15] and Pitot tubes [16]. Though these techniques are robust and proven to work, they are not ideal for measuring the velocity in TC flow. The aforementioned methods measure the magnitude of the velocity, not the individual components, and are directionally ambiguous using a single probe. Of course, one could use multiple probes [17] to obtain the flow direction. Another problem is that they alter the flow under consideration. Though this is not an issue for non-recirculating setups, like an open-ended wind tunnel, it can be a severe issue in recirculating (closed) setups, *e.g.* a TC apparatus, a rotating drum, or a Rayleigh–Bénard cell [18]. For a large range of Reynolds numbers it is known that vortices will be shed [19] from these probes, either in the form of a Kármán vortex street or as a turbulent wake, depending on the geometry and Reynolds number. These vortices can survive a full revolution, which has been observed in rotating drum experiments [20].

The TC setup used in the present work, the Twente Turbulent Taylor–Couette (T^3C) [21–23], distincts itself from other setups by many features: variable gap and radius ratio, precise temperature control, independently rotatable cylinders, and a fully optically accessible gap. The outer cylinder is constructed from 2.5 cm thick PMMA (*Poly-(methyl methacrylate)*), which enables optical measurement techniques, *e.g.* Particle Tracking Velocimetry (PTV) [24,25], Particle Imaging Velocimetry (PIV) [26,27], and Laser Doppler Anemometry (LDA) [28,29]. These methods, by their very nature, will not disturb the flow under consideration. In addition these techniques are able to measure the velocity components and are directionally sensitive, such that they are capable of detecting flow reversals. The addition of seed particles is imperative for these techniques, and one should check if these particles accurately reflect the velocity of the flow, as discussed below. Additionally, particles should not change the dynamics of the flow, in particular, some particles act as a surfactant in two-phase flows [30–32].

2. Laser Doppler anemometry

LDA is based on the Doppler effect. The most common version of LDA, is a so-called dual beam heterodyne configuration [29], see Figs. 2 and 3. In this configuration two beams are crossed and focused in the flow, creating an interference pattern. Seed particles, added to the flow, passing through the interference pattern will

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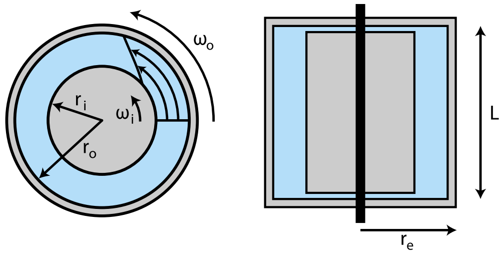

Fig. 1. *Left:* Top view of TC apparatus, two concentric cylinders are rotating. Control parameters are the rotation rates ω_i and ω_o, where the subscripts denote inner cylinder and outer cylinder, respectively. The inner cylinder has a radius of $r_i = 20$ cm, and the outer cylinder has an inner-radius of $r_o = 28$ cm. *Right:* Vertical cross section of a TC apparatus. The outer cylinder has an outer-radius of $r_e = 30.5$ cm and a height $L = 92.7$ cm. The outer cylinder is made from optically transparent PMMA (*Poly-(methyl methacrylate)*), and is attached to the top and bottom end plates.

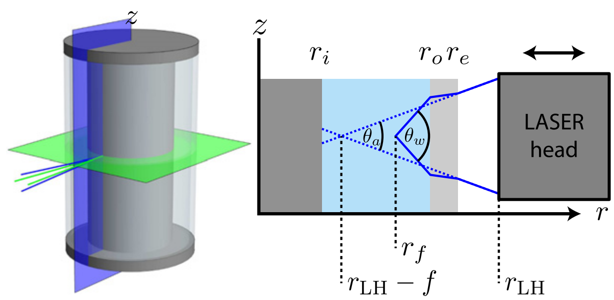

Fig. 2. *Left:* The azimuthal and axial components of the velocity are measured, laser beams are in the green and blue planes respectively. *Right:* Vertical cross section showing two laser beams. The dashed lines are beams *without* refraction, the angle between the beams is denoted θ_a, where a stands for air. The solid lines are beams *with* refraction, θ_w is the angle between the beams in water. r_{LH} is the position of the laser head and $r_{\mathrm{LH}} - f$ is position of the focus without refraction, while r_f is the real position of the focus. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

scatter light with a specific frequency. This light is then captured by a photo detector and converted to a current from which the Doppler shift can be calculated. Knowing the optical geometry of the setup one can directly calculate the velocity from the Doppler shift [29]:

$$f_d = \frac{2\sin(\theta/2)}{\lambda}|v_k| \tag{1}$$

where f_d is the Doppler shift, λ the wavelength of the laser, θ the angle between the beams, and v_k the component of the velocity along $\mathbf{k}_1 - \mathbf{k}_2$, where $\mathbf{k}_i$ are the propagation vectors of the laser beams. To add directional sensitivity one has to frequency shift one of the beams, accomplished by a Bragg cell. More details about the Bragg cell, the fringe-model, and LDA in general, can be found in *e.g.* Refs. [29,33].

Laser Doppler anemometry is a so-called absolute measurement method and therefore does not require calibration against a known flow. This, however, does not mean that a measurement of velocity is error free. Any misalignment in the optical arrangement, and any imperfection in the lenses (*e.g.* astigmatism [34]) will cause errors. In addition, any particle traveling through the beams prior to focussing can have adverse effects on the formation of a well-defined measurement volume. Similarly, any spatial inhomogeneity of the refractive index causes the focal point to shift, and the waists to mismatch in the measurement volume [35]. Furthermore, the number of particles in the interference zone fluctuates; particles move in and out the measurement volume and induce noise in the collected signal.

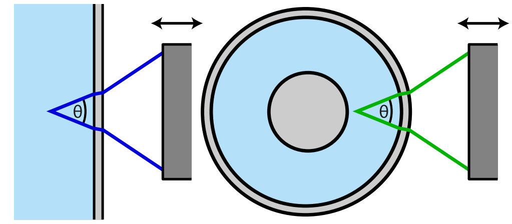

Fig. 3. *Left:* Typical geometry of LDA, equivalent to the vertical plane in the current application. The beams are passing through flat interfaces, and θ does not vary with laser-head position. *Right:* Horizontal plane: laser beams are affected by the curved interfaces, and therefore θ is a function of radial position.

2.1. Curvature effects

In most LDA applications the laser beams travel through flat surfaces, see Fig. 3. In this case, Eq. (1) can be simplified by invoking Snell's law:

$$\frac{f_d}{2|v_k|} = \frac{\sin(\theta_w/2)}{\lambda_w} = \frac{\sin(\theta_a/2)}{\lambda_a} \tag{2}$$

where quantities with a subscripts denote quantities in air, and w in water. Eq. (2) is only applicable if the interfaces are flat and the optical axis is perpendicular to those interfaces; it is only then that $\theta_a/2$ is the angle of incidence and $\theta_w/2$ the angle of refraction. The difference in refractive index is absorbed by the changing wavelength. So for the case of flat interfaces, θ_a can be obtained from the focal length and the beam separation, and together with λ_a, given by the laser, the velocity can be calculated from the Doppler shift (Eq. (2)). Note that the refractive indices of the container and water are irrelevant; they are not used in the calculation of the velocity.

For the case of a curved surface, see Fig. 3, Eq. (2) does not hold. For this case Snell's law cannot (easily) be applied in order to transform θ_w to θ_a. A prerequisite of calculating the correct velocity is therefore the knowledge of θ_w as a function of gap-position.

Most commonly the calculation of the velocity is implemented in the supplied software and implicitly assumes Eq. (2) to hold. For the case of curved interfaces this equation does not hold, and therefore the measured velocity has to be corrected by multiplying it with a correction factor C_θ:

$$C_\theta = \frac{u_{\phi,\mathrm{real}}}{u_{\phi,\mathrm{measured}}} = \frac{n_a\sin(\theta_a/2)}{n_w\sin(\theta_w/2)}, \tag{3}$$

where a subscripts denote quantities in air, and w in water; see also Fig. 2.

3. Solutions

The problem at hand is predominantly solved by mounting prisms (see *e.g.* [36]) to the outer cylinder of the TC apparatus, or by putting the entire apparatus inside a liquid bath (see *e.g.* [37]) with flat windows. The latter has two purposes: the liquid bath can act as a coolant and match the refractive index of the working fluid. In this way the beams travel through the outer cylinder with less deflection; this solution is, however, not perfect because of the finite thickness of the outer cylinder. Matching the refractive indices of the working fluid, the liquid bath, and the outer cylinder does solve the issue, but becomes cumbersome for large scale devices, or impossible if the studied fluid is a gas. The use of prisms is tantamount to the use of a liquid bath, and is also unable to fully correct for the problem. Furthermore, applying prisms is technically demanding once the outside is in motion. Theoretically one can derive the trajectories of the laser light. Ref. [38] derives these trajectories and even finds simplifications